education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1
SEPTEMBER 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x-6)(x+5)=0$ (2)

1.1.2 $7x^2-11x+3=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 \geq 5x$ (4)

1.1.4 $3\sqrt{x+12}-x=8$ (5)

1.2 Solve for x and y simultaneously:

$2y=5+x$ and $y^2+3xy = 2x^2+50$ (6)

1.3 Determine the value of: $\dfrac{\left(2^{2p-1}\right)^3}{\sqrt{7^k}}$ if $2^{6p}=81$ and $7^k=729$ (4)

[24]

QUESTION 2

Consider the linear pattern: 4; 10; 16; …

2.1 Write down the value of the following term of the pattern. (1)

2.2 Determine the value of the 50^{th} term of this pattern. (2)

2.3 A quadratic sequence is defined as: $P_k = \sum_{n=0}^{k-1}(6n-2)$

2.3.1 Show that the first 3 terms of the quadratic sequence are given by:
$-2; 2; 12; ...$ (3)

2.3.2 Determine the general term (P_k) of the quadratic sequence. Write your answer in the form $P_k = ak^2+bk+c$. (4)

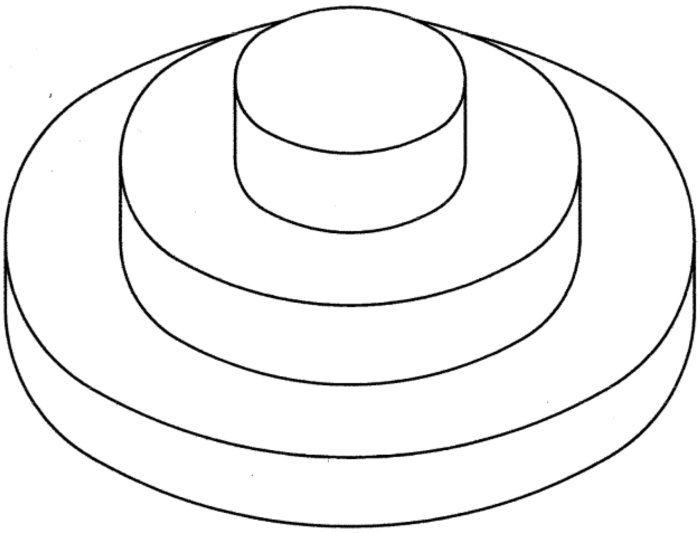
2.3.3 Determine the value of the 50^{th} term of this quadratic sequence. (2)

2.3.4 The number of terms that must be added to P_{50} to form P_q is m. The difference between P_{50} and P_q of the quadratic sequence is 7 920 + m. Determine m. (5)

[17]

QUESTION 3

A sightseeing point is built by placing concrete cylinders with a height of 0,2 m on top of one another. The radius of each consecutive cylinder is $\frac{4}{5}$ of the previous cylinder. The radius of the cylinder at the bottom is 15 m.



3.1 John is standing on the 17th cylinder. Calculate John's height above the ground. (1)

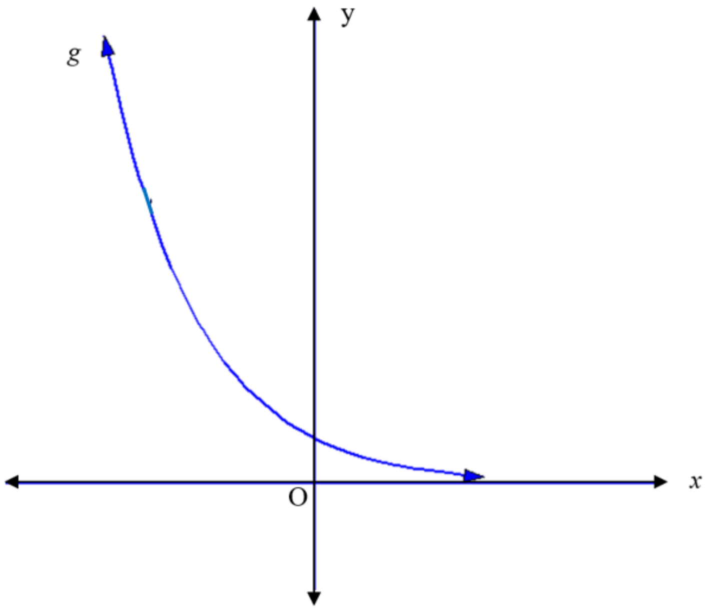
3.2 Calculate the volume of the 17th cylinder. (3)

3.3 Calculate the volume of concrete that will be used to fill the first 17 cylinders. (4)
[8]

QUESTION 4

Sketched below is the graph of $g(x)=\left(\frac{2}{5}\right)^{x}$.

- A(p ; 0,59) is the point of intersection of g(x) and $g^{-1}(x)$.
- B(-2 ; q) is a point on $g(x)$.



4.1 Calculate the value of q. (2)

4.2 Write down the equation of $g^{-1}(x)$ in the form $y = \ldots$ (2)

4.3 Write down the domain of $y = g^{-1}(x)$. (2)

4.4 For which values of x will: $g(x) \leq g^{-1}(x)$. (2)

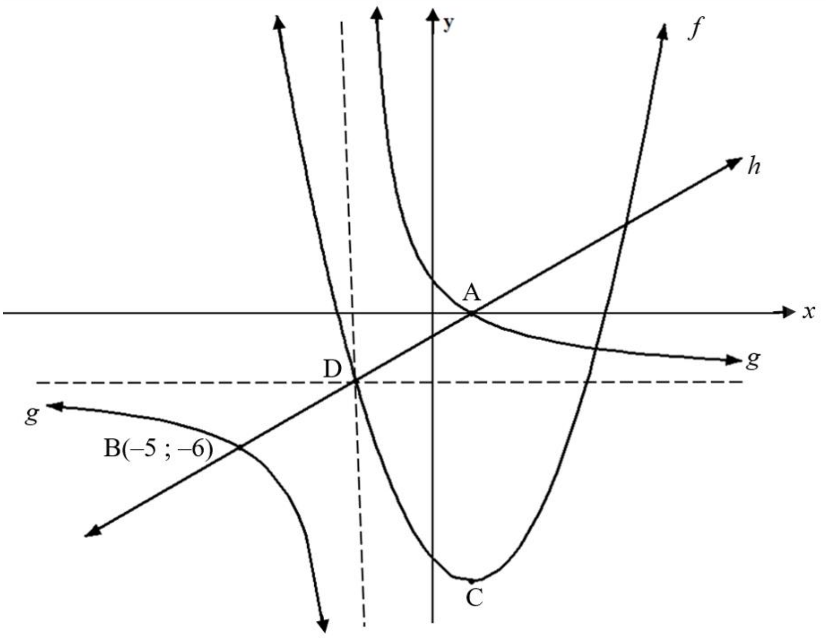
4.5 Describe the translation from g to $k(x)=\left(\frac{5}{2}\right)^{-x+2}-\frac{5}{2}$. (3)

[11]

QUESTION 5

The graphs of $g(x) = \dfrac{a}{x+r} + t$ and $f(x) = (x+p)^2 + q$ are sketched below.

- The line $h(x) = x - 1$ is an axis of symmetry of g.
- Point A is the x-intercept of g.
- A and B(-5 ; -6) are the points of intersection of g and h.
- The axis of symmetry of f intersects the x-axis at A.
- C is the turning point of f.
- D, a point on f, is the point of intersection of the asymptotes of g.



5.1 Determine the coordinates of A. (2)

5.2 Show that the coordinates of D is given by: D$(-2; -3)$ (2)

5.3 Determine the equation of g. (3)

5.4 Show that the equation of f is: $f(x) = x^2 - 2x - 11$. (3)

5.5 Determine the x-intercepts of f. (3)

5.6 For which values of x will $f^{/}(x).f(x) < 0$? (2)

5.7 Calculate the maximum value of $\dfrac{f^{//}(x)}{f(x)+14}$. (3)

5.8 For which value(s) of m will $(x+m)^2 - 2(x+m) - 11 = x - 1$ have TWO different negative roots? (7)

[25]

QUESTION 6

Frits deposited R3 000 into a savings account at the end of January 2004. He continued to make monthly deposits of R3 000 at the end of each month up to the end of December 2023. The savings account earned interest of 7,5% per annum, compounded monthly.

6.1 Calculate how much money will be in the account on 31 December 2023. (4)

6.2 Two years after Frits opened the savings account, he decided to invest Rx of his bonus each year at the end of the year to boost his savings account. He made his last deposit of Rx two years before 31 December 2023.

6.2.1 Calculate the yearly effective interest rate on his investment. Give your answer correct to 4 decimal places. (3)

6.2.2 Calculate his yearly deposit of Rx if he wants R3 500 000 in his savings account on 31 December 2023. (6)

[13]

QUESTION 7

7.1 Given: $f(x) = 5x^2 + 2x$

Determine $f'(x)$ from first principles. (5)

7.2 Determine $f'(x)$ if:

7.2.1 $f(x) = 5x^4 - x^3 + 2x$ (3)

7.2.2 $f(x) = \dfrac{8x^{\frac{1}{2}} + 4}{2x^3}$ (4)

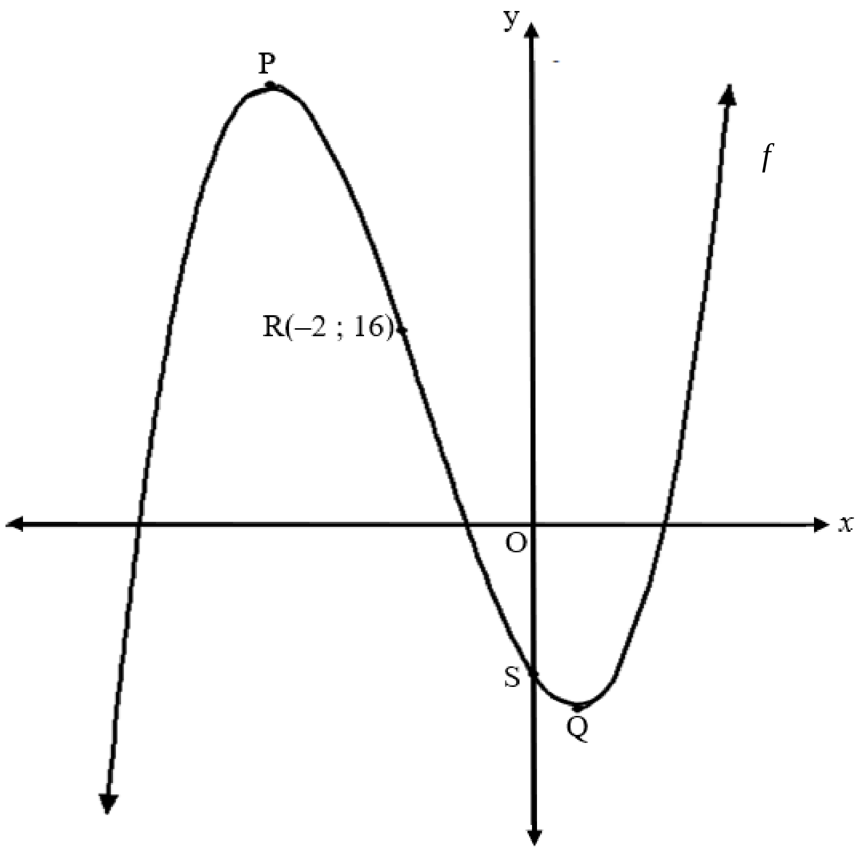
7.3 If $y = 4x^2 - 3$ and $2xt = 7$.

Determine $\dfrac{dy}{dt}$. (4)

[16]

QUESTION 8

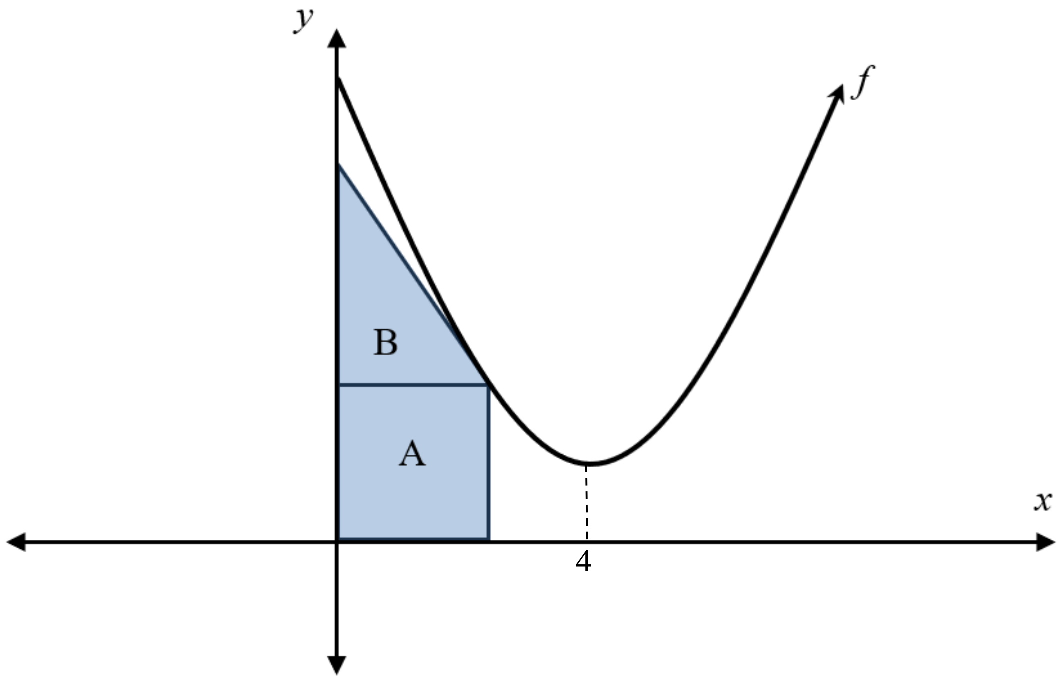
The graph of $f(x) = x^3 + ax^2 + bx + c$ is drawn below. The line $g(x) = -16x + k$ is a tangent to f at R(– 2 ; 16). Graph f is concave up at $x > -\frac{5}{3}$. P and Q are the turning points of f. S is the y-intercept of f.



8.1	Show that $a = 5$; $b = -8$ and $c = -12$.	(5)
8.2	Determine the coordinates of P and Q.	(3)
8.3	Sketch the graph of f'. Clearly indicate the x-intercepts and the x-value of the turning point(s).	(3)
		[11]

QUESTION 9

A river boards the farm of a farmer, which is represented by the equation $f(x) = x^2 - 8x + 17$. A tarred road is represented by the x- and y-axes and a border fence at $x = 4$.



9.1 Show that the area of the rectangular field (shaded area **A**), is given by: (1)

$$\text{A}(x) = x^3 - 8x^2 + 17x.$$

9.2 Determine the area of the largest rectangular field that the farmer can fence in (shaded area **A**). (5)

9.3 He decided to include an additional triangular field (shaded area **B**). Determine the largest area of the triangular field, if the base of area B is the same as the base of area A.
(Note: The fence should only touch the river and not cross it.) (4)
[10]

QUESTION 10

Tom, Dic and Harry are friends and are learners at the same high school. All three are in the same mathematics class. Some days they are absent from the mathematics class. The probability that neither Tom nor Harry is absent from the mathematics class on a specific day, is 0,42. The probability that Tom is absent from the mathematics class on a randomly selected day is 0,40.

10.1 Calculate the probability that Tom or Harry will be absent from the mathematics class on a random selected day. (1)

10.2 The mathematics teacher was suspicious about the absenteeism of Tom and Harry from the mathematics class. He investigated and realised that their absenteeism is independent from one another.

Determine the probability that Tom and Harry will be absent from the mathematics class on the same day. (4)

10.3 Calculate the probability that only Tom will be absent from the mathematics class on a random selected day. (1)

10.4 The mathematics teacher finds out that the probability that Harry and Dic will be absent from the mathematics class on a random selected day is 0,16 and that the probability that Harry or Dic will be absent is 0,3.

Calculate the probability that Dic will be absent from the mathematics class on a randomly selected day. (2)

10.5 Will there be a day that Dic is absent from the Mathematics class, but Harry is present? (1)

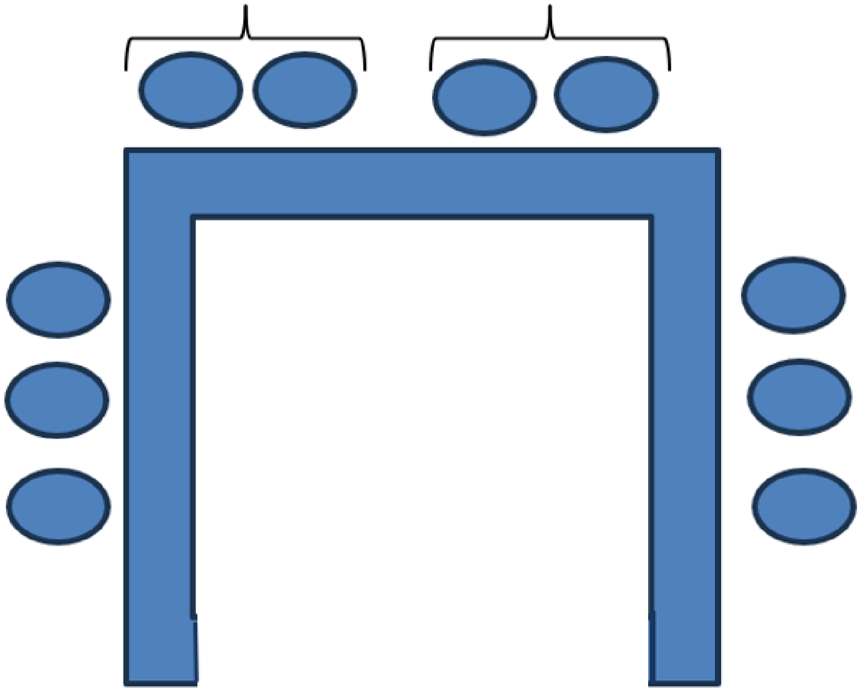
[9]

QUESTION 11

Two learners (one boy and one girl) from each grade (grade 8, 9, 10, 11 and 12) are elected to form the grade representatives of the high school.

11.1 When meeting, they start and end with a prayer. In how many different ways can they select somebody to start and someone else to end the meeting with a prayer? (1)

11.2 When they meet, they sit at a u-shaped table with the two grade 12 members next to each other and the two grade 11 members next to one another at the top of the table. The rest of the members sit on the remaining chairs in any order.



Determine in how many different ways can the members be arranged along the u-shaped table? (Refer to the diagram.) (2)

10.3 They decide that during assembly on a Monday, they will be seated on the first chair of each row, for the first 10 rows. If they are randomly allocated seats, determine the probability that a boy will be seated in the first row, and another boy will be seated in the tenth row. (3)

[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$