education

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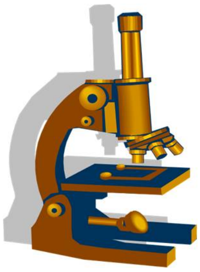
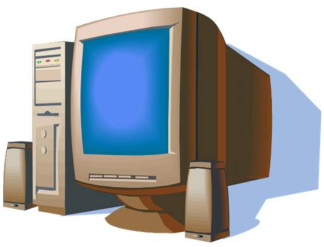
NORTH WEST PROVINCE

2020 MATRIC



MATHEMATICS PAPER 1

COLLECTABLE MARKS



ALGEBRAIC EXPRESSIONS, EQUATIONS AND INEQUALITIES			
	ATP: GARDE 10	**ATP: GRADE 11**	
	Rational and irrational numbers	Theory of numbers	
	Linear equations		
	Quadratic equations	Solve quadratic equations by Factorisation Taking square roots Completing the square Using quadratic formula	
	Literal equations (changing subject of the formula)		
	Linear inequalities (interpret solutions graphically)	Quadratic inequalities (interpret solutions graphically)	
	System of linear equations	Equations in two unknowns One equation linear and the other quadratic	
		Nature of roots	
	Word problems	Word problems (modelling)	
	Exponential laws and simplify using (rational exponents)	Simplify expressions using laws of exponents Add, subtract, multiply and divide surds	
	Exponential equations	Solve equations using laws of exponents Solve simple equations involving surds	

VOCABULARY			
	Expression	It is made up of constants, variables and number operations It only be SIMPLIFIED (write it in simple form) by grouping, adding, subtracting like terms or factorising.	
	Equations	Expressions with equal sign. Main question is SOLVE When solving equations, you need to find the unknown values	
	Quadratic formula	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	
	Discriminant	$\Delta=b^2-4ac$ where $y=ax^2+bx+c$	
	Perfect square	A number that is the square of an integer	
	Standard form	Linear: $ax+by=c$ Quadratic: $ax^2+bx+c=0$	

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+1)=6$ (3)

1.1.2 $3x^2+4x=8$ (correct to TWO decimal places.) (4)

1.1.3 $4x^2+1\geq 5x$ (4)

1.1.4 $\sqrt{2x+3}-x=0$ (4)

1.2 Solve for x and y simultaneously: (6)

$y+2x-3=0$ and $x^2+y+x=y^2$

[21]

QUESTION2 NW SEP 2015

2.1 Solve for x:

2.1.1 $(x-1)(x+8)=10$ (4)

2.1.2 $4x+\frac{4}{x}+11=0;\ x\neq 0$

(Leave your answer correct to TWO decimal places.) (4)

2.1.3 $6x<3x^2$ (5)

2.2 Solve for x and y simultaneously:

$3+x=2y$ and $x^2+4y^2=2xy+7$ (7)

2.3 For which values of m will $x+y$ be a factor of x^m+y^m? (2)

[22]

QUESTION 3 NW SEP 2016

3.1 Solve for x:

3.1.1 $7x(2x-1)=0$ (2)

3.1.2 $2x^2+x=4$ (Leave your answer correct to TWO decimal places.) (4)

3.1.3 $(x-4)(x+5)\geq 0$ (3)

3.1.4 $3x^{\frac{2}{5}}-5x^{\frac{1}{5}}-2=0$ (4)

3.2 Solve for x and y simultaneously:

$\frac{2x}{1+y}=1;\ y\neq -1$ and $(3x-y)(x+y)=0$ (6)

3.3 Given: $f(x)=\frac{3}{x-2}$ and $g(x)=3^{x-2}$. Explain why $f(x)=g(x)$ will have only ONE root. Motivate your answer. (3)

[22]

QUESTION 4 NW SEP 2017

4.1 Solve for x:

4.1.1 $x^2-5x=6$ (3)

4.1.2 $2x^2+8x-3=0$ (correct to TWO decimal places.) (4)

4.1.3 $x^2-64\leq 0$ (2)

4.1.4 $4^x-8.2^x=0$ (4)

4.2 Solve for x and y simultaneously:

$2^{x+y}=4$ and $x^2=52-y^2$ (7)

(West Coast ED – Sep 2014)

4.3 Given: $2mx^2=3x-8$, where $m\neq 0$. Determinethe value of m if the roots of the given equation are non-real. (4)

4.4 If $i=\sqrt{-1}$, show WITHOUT THE USE OF A CALCULATOR, that $(1+i)^{24}=4096$. (4)

[28]

QUESTION 5 NW SEP 2018

5.1 Solve for x:

5.1.1 $2x(5x-3)=0$ (2)

5.1.2 $-x^2+4=5x$ (Leave your answer correct to TWO decimal places.) (4)

5.1.3 $\sqrt{x-6}-2=\dfrac{15}{\sqrt{x-6}}$ (5)

5.1.4 $\left(x^2+2\right)\left(x-3\right)<0$ (2)

5.2 Solve for x and y simultaneously: (6)

$x+2y=3$ and $3x^2+4xy+9y^2-16=0$

5.3 Determine the sum of the digits of: $2^{2022}.5^{2018}$ (4)

[23]

QUESTION 6 NW SEP 2019

6.1 Solve for x:

6.1.1 $3x^2-18x=0$ (3)

6.1.2 $7x^2-4x=5$ (Leave your answer correct to TWO decimal places.) (4)

6.1.3 $\left(x+5\right)\left(x-2\right)>0$ (2)

6.1.4 $26-5^{2x}=\left(5^x-6\right)^2$ (6)

6.2 Solve for x and y simultaneously: (6)

$x-4y=5$ and $3x^2-5xy+2y^2=25$

6.3 Solve for x if: $x=\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12+...}}}}$ (4)

[25]

PATTERNS, SEQUENCE AND SERIES		
ATP: GRADE 10	**ATP: GARDE 11**	**ATP: GRADE 12**
Linear patterns	Linear patterns	Number patterns dealt with in grade 10 and 11

Determine general term using $T_n = dn + q$	Determine general term using $T_n = dn + q$	Patterns, including arithmetic sequences and series
	Exponential patterns	Sigma notation
	Number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore quadratic. $T_n = an^2 + bn + c$	Derivation and application for the sum of arithmetic series $S_n = a + (a+d) + (a+2d) \ldots a + (n-1)d$ $\vdots \quad \vdots$ $\therefore S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
	$a + b + c =$ First term of quadratic sequence	Number patterns, including geometric sequences and series
	$3a + b =$ First term from the row of first difference	Derivation and application of the formula for the sum of geometric series $S_n = a + ar + ar^2 + .. + ar^{n-1}$ $\vdots \quad \vdots$ $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$
	$2a =$ Second difference	Sigma notation
		Infinite geometric series

VOCABULARY	DEFINITION/ CLARIFICATION
First term or Leading term	a or T_1
Common difference	$d = T_n - T_{n-1}$ Test $T_2 - T_1 = T_3 - T_2$
Which term or How many terms	n or n^{th} term
Arithmetic sequence	Each term is the result of adding the same number (called the common difference) to the previous term. $T_n = a + (n-1)d$
Common ratio	$r = \dfrac{T_n}{T_{n-1}}$ Test $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2}$
Arithmetic series	$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ or $S_n = \dfrac{n}{2}(a + \ell)$ where $\ell = T_n$
Geometric sequence	Each term of G. P is found by multiplying the previous term by a fixed number (called common ratio). $T_n = a.r^{n-1}$
Geometric series	$S_n = \dfrac{a(r^n - 1)}{r-1}$ or $S_n = \dfrac{a(1-r^n)}{1-r}$
Infinite series	$S_\infty = \dfrac{a}{1-r}$, $-1 < r < 1$
Quadratic sequence	The sequence whose second difference is constant: $T_n = an^2 + bn + c$
Sigma notation	The sum of $\sum_{k=1}^{n} T_n$
Find T_n given S_n	$T_n = S_n - S_{n-1}$

QUESTION 7

7.1 The fifth term of an arithmetic sequence is zero and the thirteenth term is equal to 16.
Determine:

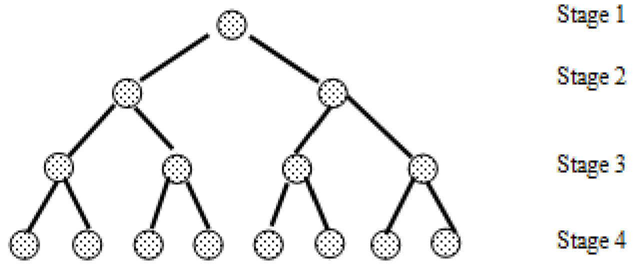
7.1.1 The common difference (4)

7.1.2 The first term (2)

7.1.3 The sum of the first 21 terms (4)

7.2 Determine the twelfth term of the geometric sequence $3;\ 1;\ \dfrac{1}{3};\ \ldots$ (4)

7.3 Cells are continually dividing and thus increasing in number. A cell divides and becomes two new cells. The process repeats itself forming a geometric sequence. The following sketch represents this cell division.



How many cells will there be altogether after twenty stages? (4)

[18]

QUESTION 8

8.1 The following arithmetic sequence is given: $-1; 6; 13; ...$
Determine:

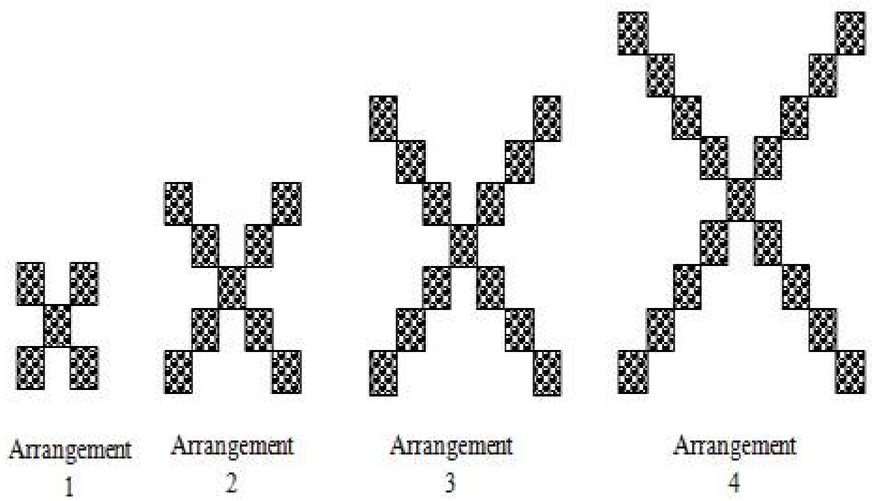
8.1.1 The 49$^{\text{th}}$ term (3)

8.1.2 The sum of the first 87 terms (3)

8.2 $20;\ 16;\ ...$ is a geometric sequence.
Calculate the sum of the first ten terms. (4)

8.3 The following are the consecutive terms of a geometric sequence:
$3x-2;\ 2x+2;\ 4x+1$ (x is a natural number) (6)
Calculate the value of x.

8.4 The tiles are arranged as shown below. The first arrangement has 5 tiles, the second arrangement has 9 tiles, the third arrangement has 13 tiles and the fourth arrangement has 17 tiles. The arrangements continue in this pattern



Derive, in terms of n, a formula for one of tiles in the n^{th} arrangement. (3)

[19]

QUESTION 9

9.1 The following arithmetic series is given: $5+9+13+...+401$.

Calculate:

9.1.1 The number of terms in the series (4)

9.1.2 The sum of the terms in the series (3)

9.2 Given the sequence: $2;\ x;\ 18;\ ...$ (4)

Calculate x if this sequence is:

9.2.1 An arithmetic sequence (3)

9.2.2 A geometric sequence (4)

9.3 Given: $\sum_{k=1}^{10} 3\left(2^{1+k}\right)$

9.3.1 Write down the first three terms of the series (3)

9.3.2 Determine the sum of the series (5)

[22]

QUESTION 10

10.1 The sequence $3;\ 9;\ 17;\ 27;\ ...$ is quadratic.

10.1.1 Determine an expression for the $n-$th term of the sequence. (4)

10.1.2 What is the value of the first term of the sequence that is greater than 269? (4)

10.2 The sum of n terms in a sequence is given by $S_n = -n^2 + 5$. (3)

Determine the 23rd term.

10.3 **(NW SEP 2016)**

$54;\ x;\ 6$ are the first three terms of a geometric sequence.

10.3.1 Calculate x (2)

10.2.2 Is this geometric sequence convergent? Motivate your answer by clearly showing all your calculations. (3)

10.4 Determine the value of k for which:

$$\sum_{r=5}^{60}(3r-4) = \sum_{p=2}^{5} k$$ (5)

[21]

QUESTION 11NW SEP 2017

11.1 $69;\ 0;\ -63;\ \ldots$ forms a quadratic sequence.

11.1.1 Write down the value of the next term in the sequence (1)

11.1.2 Calculate an expression for the n^{th} term of the sequence. (4)

11.1.3 Determine the value of the smallest term in this sequence (4)

11.2 A finite arithmetic series of 16 terms has a sum of 632.

The eleventh term if 47.

11.2.1 Calculate the 1st term of the series (5)

11.2.2 Determine the fifth term of the series (1)

11.2.3 Express the sum of the last five terms of this series in sigma notation. (2)

[17]

QUESTION 12

12.1 **NW SEP 2018**

If $x+1;\ x-1;\ 2x-5;\ \ldots$ are the first 3 terms of a geometric series, calculate:

12.1.1 The value(s) of x (5)

12.1.2 For which value of x in QUESTION 12.1.1 will the series converge? (4)

12.1.3 The sum to infinity if $x=3$ (2)

12.2 **NW SEP 2019**

Write $-4+3+10+\ldots+486$ in sigma notation. (4)

12.3 This arithmetic sequence $-11;\ -4;\ 3;\ \ldots$ forms the first three first difference of a quadratic sequence. Which term in this quadratic sequence will be the smallest? Show all your calculations. (5)

[20]

FUNCTIONS AND GRAPHS		
	GRADE 10	**GRADE 11**
	$y = ax + q$ - linear	$y = ax + q$ - linear
	$y = ax^2 + q$ - parabolic	parabolic $y = a(x+p)^2 + q$ $y = ax^2 + bx + c$ $y = a(x-x_1)(x-x_2)$
	$y = \frac{a}{x} + q$ - hyperbolic	$y = \frac{a}{x+p} + q$ - hyperbolic
	$y = ab^x + q;\ b > 0,\ b \neq 1$ - exponential	$y = a.b^{x+p} + q;\ b > 0,\ b \neq 1$ - exponential
	Point-by-point plotting	
	Generalise effects of parameters a and q on graphs	Generalise effect of $a,\ p$ and q on graphs
	Drawing the graphs using parameters, $x-$intercept(s) and $y-$intercept and asymptotes	Drawing the graphs using parameters, $x-$intercept(s) and $y-$intercept and asymptotes
	Finding equations of the functions and interpretation	Finding equations of the functions and interpretation
	Solving problems involving all functions	Solving problems involving all functions
		Average gradient between two points on a curve
ALGEBRAIC FUNCTIONS AND INVERSES		
GRADE 12		
	Determine and sketch graphs of the inverses of the functions defined by	Focus on the following characteristics
	$y = ax + q$	Domain and range Intercepts with the axes Turning point Minima and maxima Horizontal and vertical asymptotes Shape and symmetry Average gradient (average rate of change) Interval on which graph decreases/increases
	$y = ax^2$	
	$y = b^x\ (b > 0;\ b \neq 1)$ $y = b^x\ (0 < b < 1)$	
	$y = \log_b x \Leftrightarrow x = b^y\ (b > 0;\ b \neq 1)$ $y = \log_b x \Leftrightarrow x = b^y\ (0 < b < 1;\ b > 1)$	

VOCABULARY		
	Function	A relationship that assigns one output for each input value
	Gradient of a line	$m=\dfrac{y_2-y_1}{x_2-x_1}$
	Average gradient	$m=\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$
	Line of symmetry	A line that divides a figure so that the two parts of the figure are congruent. It divides a figure into mirror images
	Line symmetry	When a figure can be folded on line so that its two parts are congruent
	Reflection	When a figure is flipped across a line
	Asymptote	The line that a curve will get closer and closer without ever reaching it.
	Axis of symmetry (A. S)	$x=\dfrac{-b}{2a}$
	Turning point of f	$\left(\dfrac{-b}{2a}\,;\,f\left(\dfrac{-b}{2a}\right)\right)$
	$x-$axis	Horizontal number line
	$x-$coordinate	First number in an ordered pair
	$x-$intercept(s)	$x-$coordinate of the point where a curve crosses the $x-$axis
	$y-$axis	Vertical number line
	$y-$coordinate	Second number in an ordered pair
	$y-$intercept	$y-$coordinate of the point where a curve crosses the $y-$axis

THE COORDINATES OF THE TURNING POINT OF $f(x) = ax^2 + bx + c$ USING DEFFERENTIATION

If $f(x) = ax^2 + bx + c$ then the gradient at any point is $f'(x)$

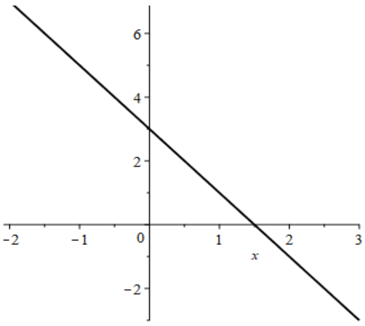
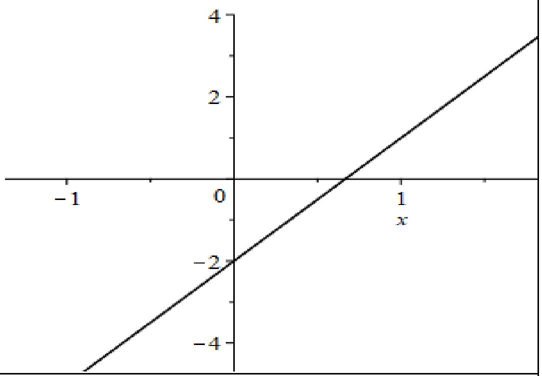
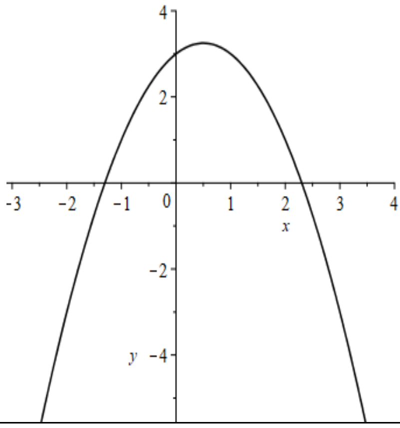
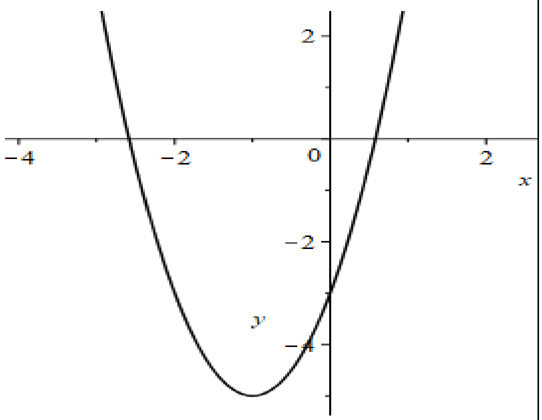
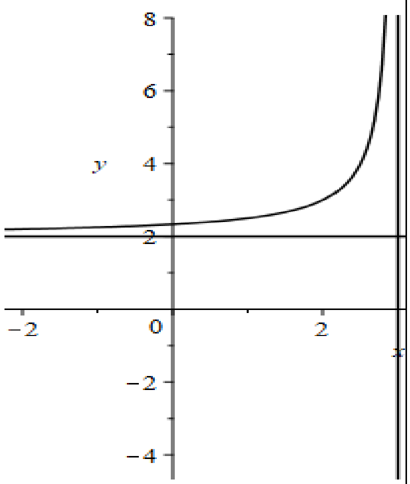
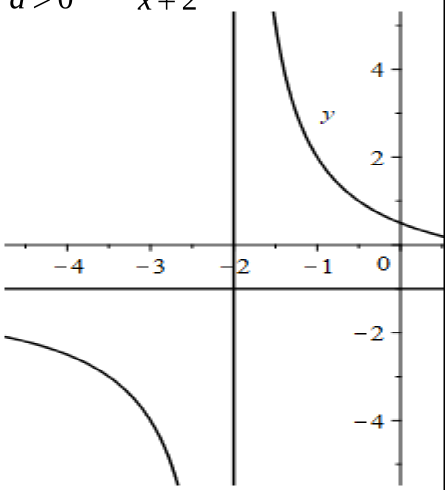
That is $f'(x) = 2ax + b$

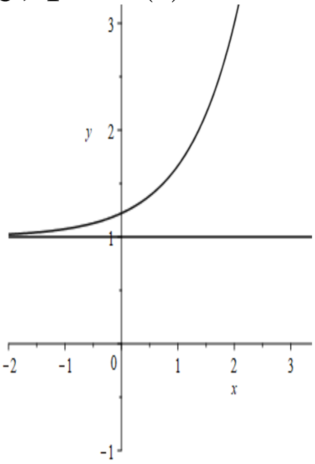
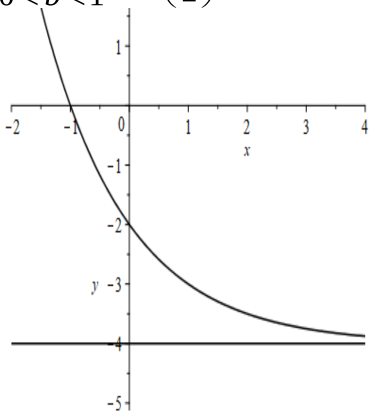
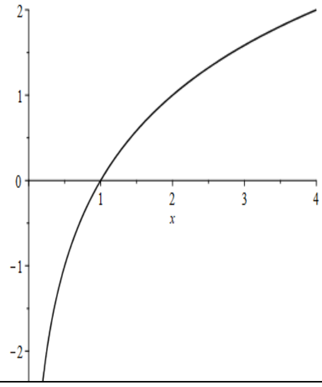
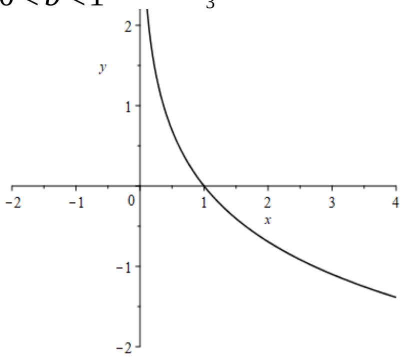
At the turning point (either maximum or minimum) $f'(x) = 0$

That is $2ax + b = 0$ and therefore

$x = \frac{-b}{2a}$

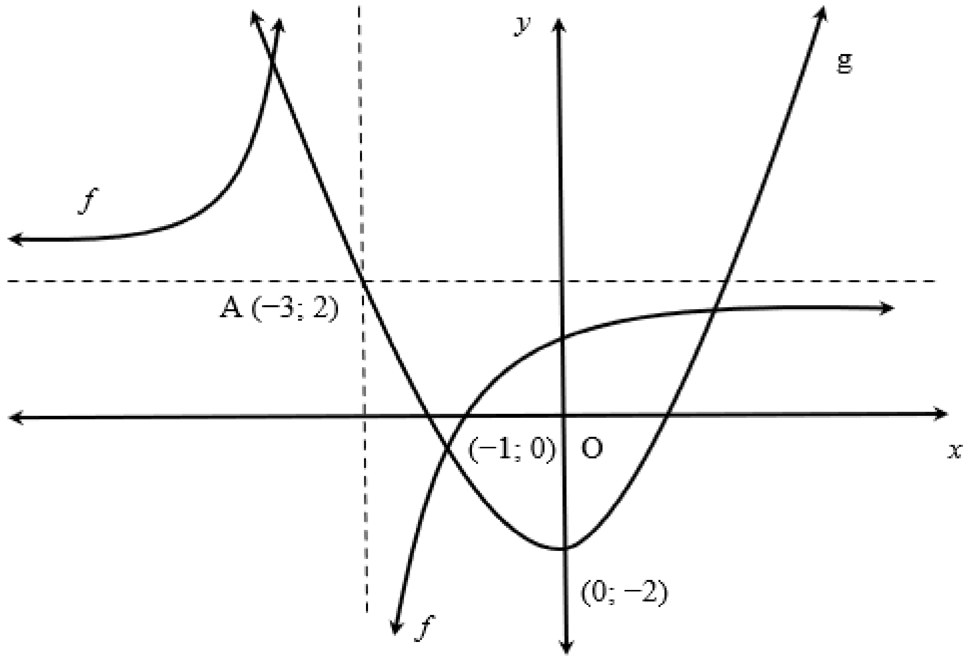
The corresponding $y-$value is $f\left(\frac{-b}{2a}\right)$

Function	Equation	Orientation	
Linear	$y = ax + q$	$a < 0$ $y = -2x + 3$ 	$a > 0$ $y = 3x - 2$ 
Parabolic	$y = ax^2 + bx + c$ $y = a(x+p)^2 + q$	$a < 0$ $y = -x^2 + x + 3$ $y = -\left(x - \frac{1}{2}\right)^2 + \frac{13}{4}$ 	$a > 0$ $y = 2x^2 + 4x - 3$ $y = 2(x+1)^2 - 5$ 
Hyperbolic	$y = \frac{a}{x+p} + q$	$a < 0$ $y = \frac{-1}{x-3} + 2$ 	$a > 0$ $y = \frac{3}{x+2} - 1$ 

Exponential	$y = a.b^{x+p} + q$	$b > 1$ $y = 2.(3)^{x+2} + 1$ 	$0 < b < 1$ $y = \left(\frac{1}{2}\right)^{x-1} - 4$ 
Logarithmic	$y = \log_b k(x+p)$	$b > 1$ $y = \log_2 x$ 	$0 < b < 1$ $y = \log_{\frac{1}{3}}(x)$ 

QUESTION 13   NW SEP 2015

The sketch below represents the graphs of: $f(x)=\dfrac{a}{x+p}+q$ and $g(x)=bx^2+c$.



The point A(−3 ; 2) is the point of intersection of the asymptotes of *f*. The graph of *f* intersects the *x*-axis at (−1 ; 0). The graph of *g* intersects the *y*-axis at (0 ; −2).

13.1 Write down the equations of the asymptotes of *f*. (2)

13.2 Determine the equation of *f*. (3)

13.3 Write down the equation of the axes of symmetry of *f* in the form $y = mx + c$ if $m < 0$. (2)

13.4 Write down the domain of $5f(x-1)$. (2)

13.5 Write down the equation of *k*, the reflection of *f* about the *y*-axis. Leave your answer in the form $y=\dfrac{a}{x+p}+q$. (2)

13.6 For which value(s) of *x* is $f(x).g'(x)\geq 0$? (2)

13.7 Determine the equation of *g*. (3)

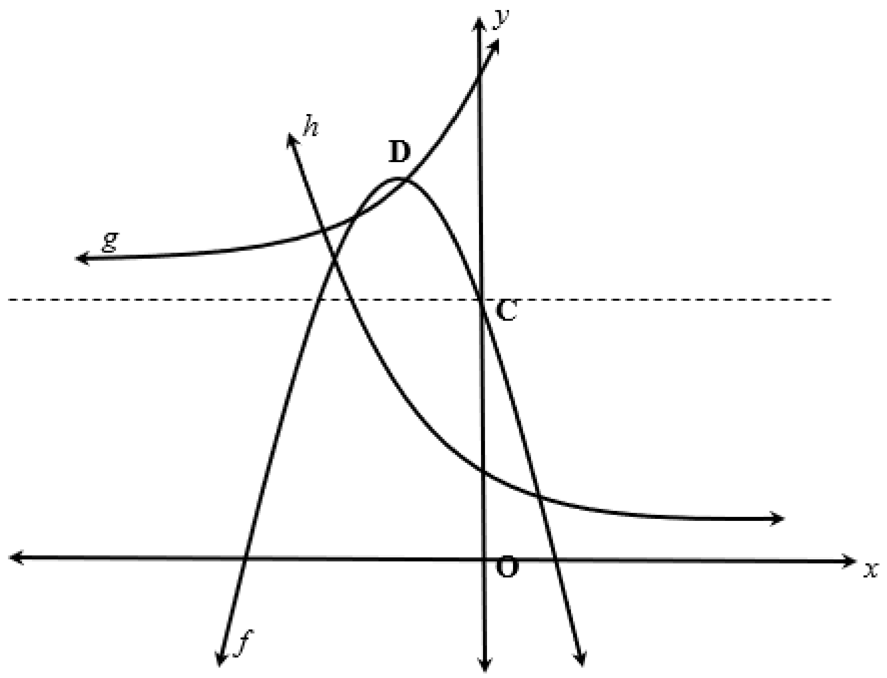
13.8 Determine the equation of $h^{-1}(x)$ if $h(x)=g(x)+2$. Leave your answer in the form $y = ...$ (3)

13.9 The inverse of *h* is not a function. Restrict the domain of *h* such that h^{-1} is a function. Sketch the restricted graph of *h* and h^{-1} on the same system of axes. (2)

[21]

QUESTION 14

The graphsof $h(x) = 3^{-x}$, $f(x) = -(x+1)^2 + 9$ and $g(x) = a.2^x + q$ are represented in the sketch below. D, the turning point of *f*, is also a point of intersection of *g* and*f*. The asymptote of *g* passes through C,the *y*-intercept of*f*.



14.1	Write down the coordinates of C.	(2)
14.2	Calculate the values of *a* and *q*.	(3)
14.3	Write down the range of *g*.	(2)
14.4	Write down the coordinates of D', if D is reflected about the line $y = 8$.	(1)
14.5	If $k(x) = (x+2)^2 + 9$, describe the transformation from *f* to*k*.	(3)
14.6	Write down the equation of $h^{-1}(x)$ in the form $y = ...$	(1)
14.7	Determine the minimum value of $y = \left(\frac{1}{3}\right)^{f(x)-5}$.	(2)

[14]

QUESTION 15 NW SEP 2016

15.1 Given: $f(x)=\dfrac{-3}{x-2}+1$

15.1 Calculate the coordinates of the $y-$intercept of $f\cdot$ (2)

15.2 Calculate the coordinates of the $x-$intercept of $f\cdot$ (2)

15.3 Sketch the graph of f in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

15.4 Write down the range of $f\cdot$ (2)

15.5 Another function $h,$ is formed by translating f, 3 units to the right and 4 units down. Write down the equation of $h.$ (2)

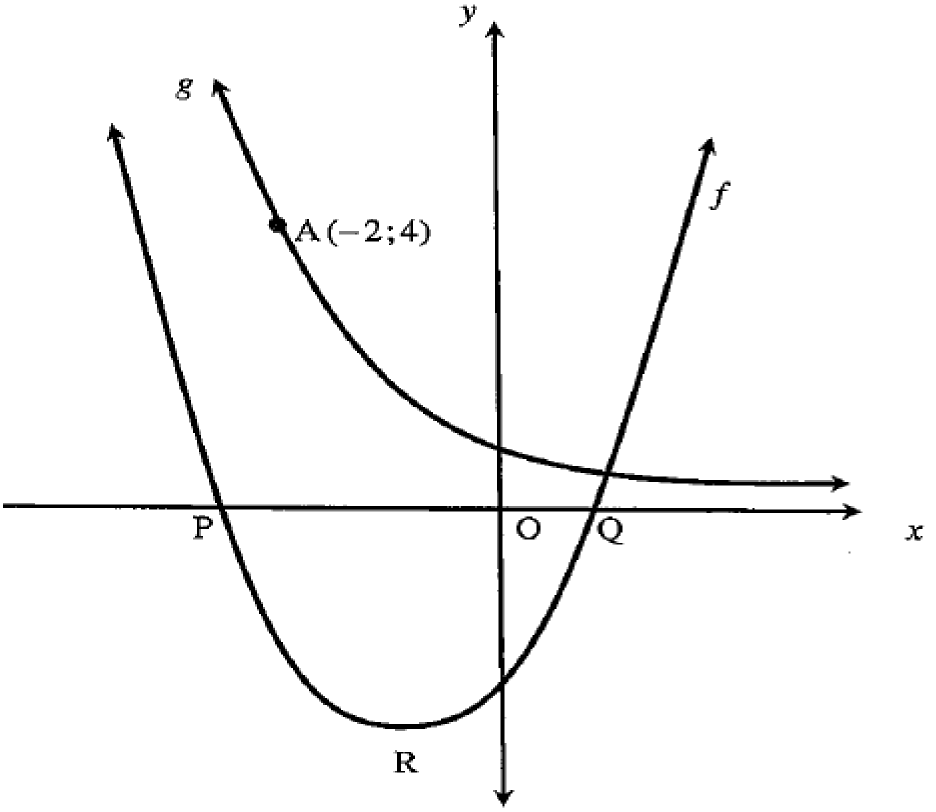
15.6 For which value(s) of x is $h(x)\leq -4$? (3)

15.7 Determine the equations of the asymptotes of $k(x)=\dfrac{3x-5}{x-1}$. (3)

[17]

QUESTION 16 NW SEP 2016

The graphs of $f(x)=2(x+1)^2-8$ and $g(x)=\left(\frac{1}{2}\right)^x$ are represented in the sketch below. P and Q are the $x-$intercepts of f and R is the turning point of f. The point $\mathrm{A}(-2;4)$ is a point in the graph of g.



16.1 Write down the equation of the axis of symmetry of f. (1)

16.2 Write down the coordinates of the turning point of f. (1)

16.3 Determine the length of PQ. (4)

16.4 Write down the equation of k, if k is the reflection of f in the $y-$axis. Give your answer in the form $y=ax^2+bx+c$. (3)

16.5 Write down the equation of g^{-1}, the inverse of g, in the form $y=...$ (1)

16.6 Sketch the graph of g^{-1} in your ANSWER BOOK, clearly showing the intercept with the axis as well as ONE other point on the graph of g^{-1}. (3)

16.7 For which value(s) of x will:

16.7.1 $g^{-1}(x)\geq -2$ (2)

16.7.2 $x.f(x)<0$ (4)

[19]

QUESTION 17 NW SEP 2017

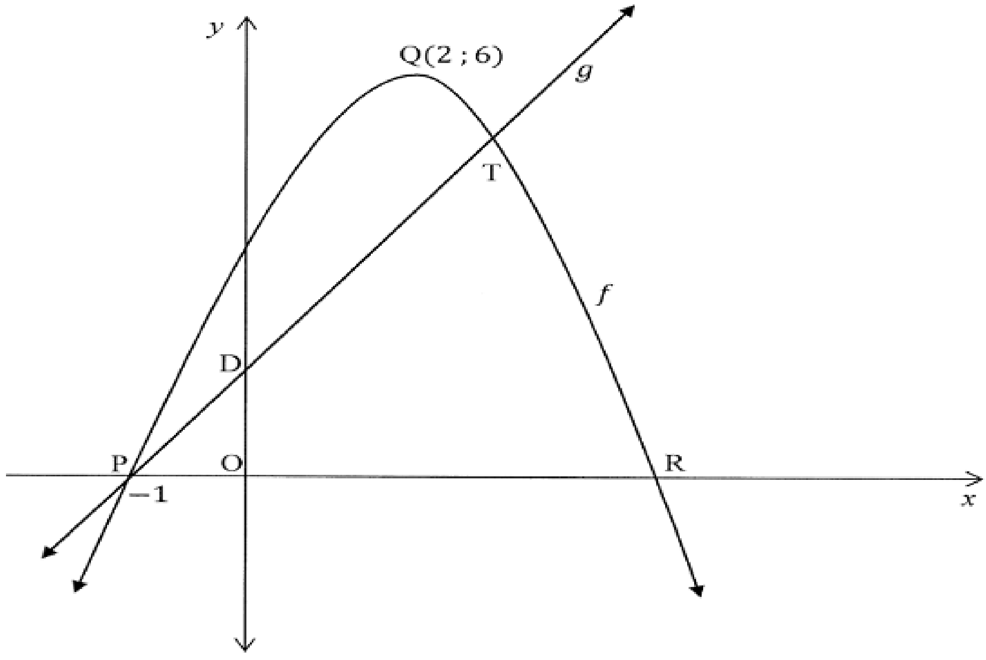
17.1 Given: $h(x)=\frac{4}{2-x}-1$

17.1.1 Write down the equation of the asymptotes of h. (2)

17.1.2 Sketch the graph of h. Show clearly all asymptotes and intercepts with the axes. (4)

17.1.3 Given: $k(x)=\frac{x+2}{x-2}$. How will you use h to sketch k? Show all your calculations. (3)

17.2 The sketch below shows the graphs of $f(x)=-\frac{2}{3}x^2+bx+c$ and g, a straight line. $\mathrm{Q}(2;6)$ is the turning point of f. P and R are the $x-$intercepts of f. D is the $y-$intercept of g. The graphs of f and g intersect at P and T.



17.2.1 Calculate the values of b and c. (3)

17.2.2 Calculate the coordinates of D if $\mathrm{PD}=\frac{\sqrt{13}}{2}$ units. (2)

17.2.3 For which value(s) of x is $x.g(x)\geq 0$? (2)

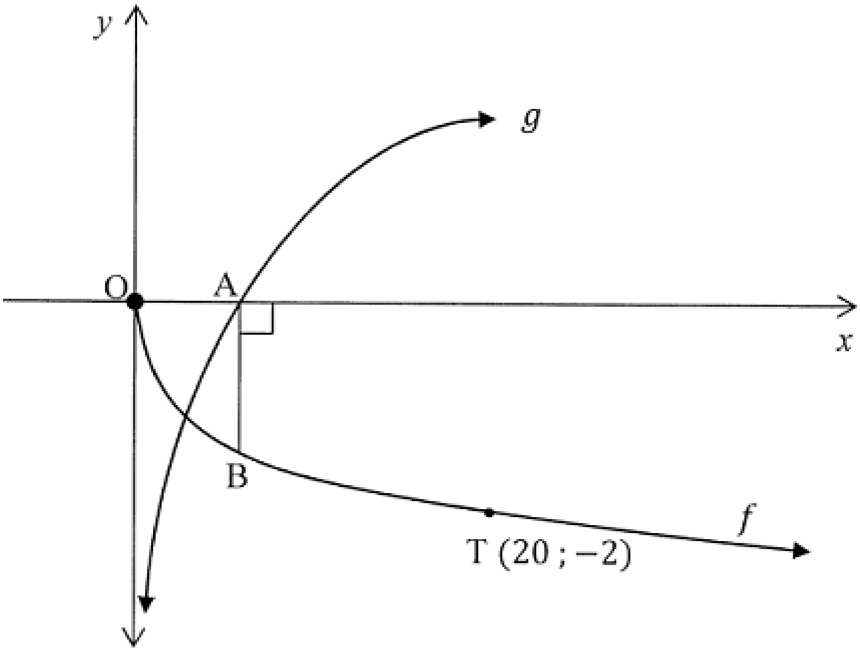
17.2.4 Determine the equation of g^{-1} in the form $y=\ldots$ (4)

[20]

QUESTION 18 NW SEP 2017

The graphs of $f(x)=-\sqrt{\dfrac{x}{5}}$ for $x\geq 0$ and $g(x)=\log_5 x$ are shown below. $\mathrm{T}(20;-2)$ is apoint on f. A is the $x-$intercept of g and B is a point f.

AB is parallel to the $y-$axis.



18.1	Write down the range of g.	(1)
18.2	Write down the coordinates of A.	(1)
18.3	Calculate the length of AB.	(3)
18.4	It is given that h is obtained when g^{-1} is shifted two units to the left. Write down the equation of h.	(3)
18.5	Determine the value(s) of x for which $f^{-1}(x)<20$.	(2)

[10]

QUESTION 19 NW SEP 2018

19.1 Given: $f(x)=\dfrac{a}{x-p}+q.$

- The point $\text{B}(-1;0)$ is an $x-$intercept of f.
- The domain of f is real numbers, but $x\neq 2$.
- The range of f is real numbers, but $y\neq 3$.
- f is a decreasing function.

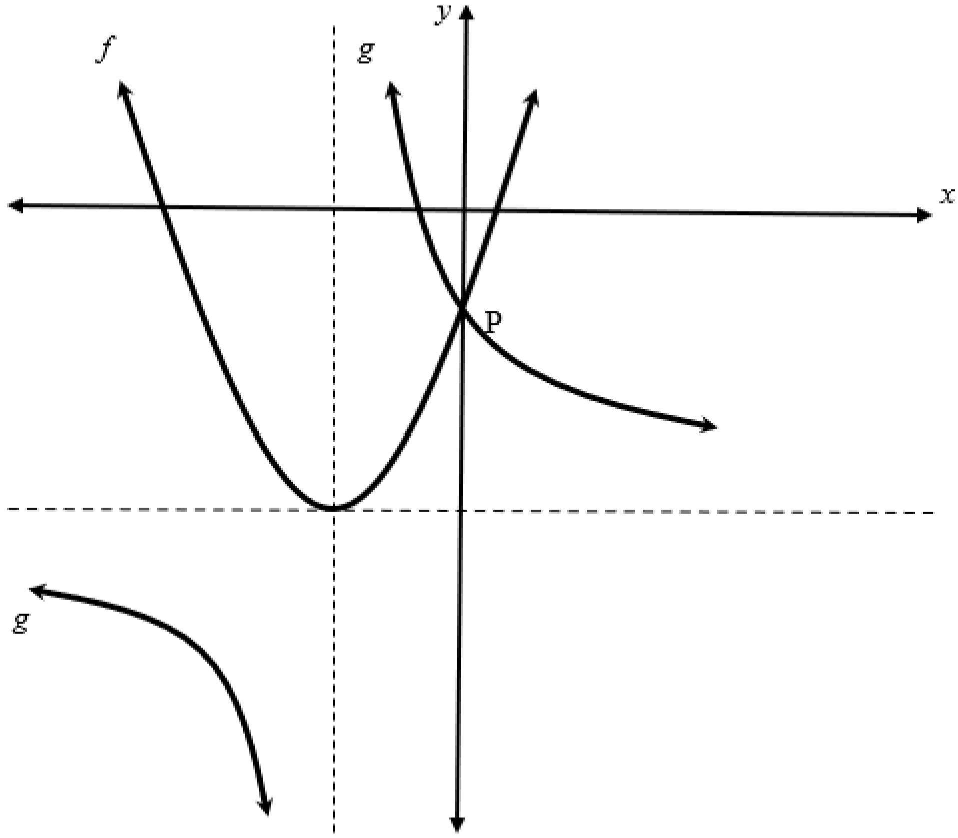
19.1.1 Determine the equation of f. (3)

19.1.2 Determine the coordinates of the $y-$intercept of f. (2)

19.1.3 Sketch the graph of f in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

19.2 The graphs of $f(x)=a(x+p)^2+q$ and $g(x)=\dfrac{2}{x+1}-3$ are sketched below.

P is the $y-$intercept of f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f.

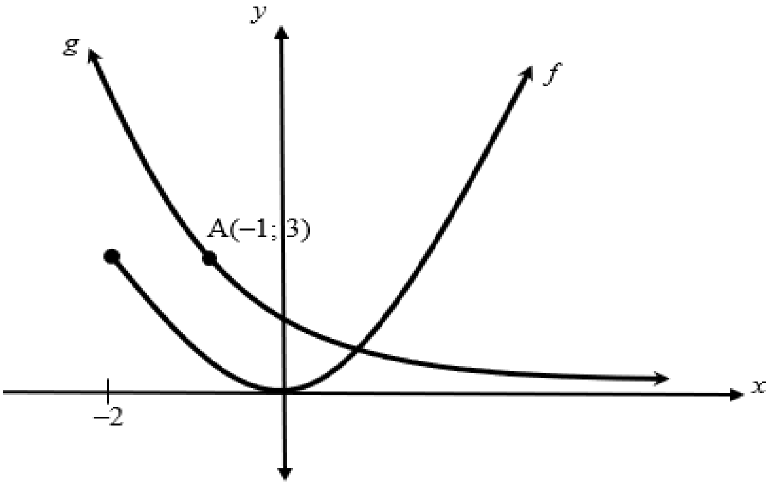


19.2.1	Write down the equation of the vertical asymptote of g.	(1)
19.2.2	Determine the coordinates of P.	(2)
19.2.3	Determine the equation of f.	(3)
19.2.4	One of the axes of symmetry of g is a decreasing function. Write down the equation of this axis of symmetry, $h(x)$.	(2)
19.2.5	For which values of k will $g(x)=h(x)+k$ have TWO real roots that are of opposite signs?	(2)
19.2.6	Give the domain of $m(x)$ is $m(x)=g(2x)+5$.	(3)

[21]

QUESTION 20 NW SEP 2018

The diagram below shows the curves of $f(x)=\frac{1}{4}x^2$, where $x \geq -2$ and $g(x)=a^x$, where $a>0$. The point $A(-1;3)$ lies on the graph of g.
P is the $y-$intercept of f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f.



20.1 Show that $g(x)=\left(\frac{1}{3}\right)^x$. (1)

20.2 For which value(s) of x is the graph of f strictly decreasing? (2)

20.3 Determine the inverse of f in the form $y=\cdots$ (2)

20.4 Sketch the graph of f^{-1} in your ANSWER BOOK. (2)

20.5 Write down the range of f^{-1}. (2)

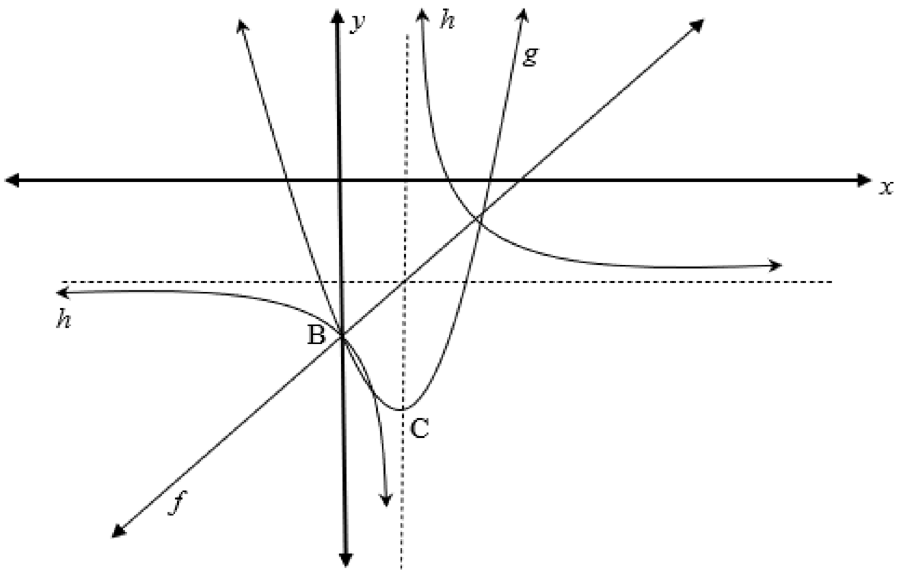
20.6 Determine the inverse of g in the form $y=\cdots$ (2)

20.7 For which values of x will $g^{-1}(x) \geq -1$? (2)

[13]

QUESTION 21 NW SEP 2019

The graphs of $g(x)=\frac{1}{2}(x-2)^2-9$ and $h(x)=\frac{a}{x+p}+q$ are sketched below. The axis of symmetry of graph g is the vertical asymptote of graph h. The line f is an axis of symmetry of graph h. B is the $y-$intercept of h, g and f.



21.1 Write down the coordinates of C, the turning point of g. (2)

21.2 Determine the coordinates of B. (2)

21.3 Write down the equation of f. (2)

21.4 Determine the equation of h. (5)

21.5 Write down the equationsof the vertical and horizontal asymptotes of $k(x)=3h(x)-2$. (2)

21.6 Determine the $x-$intercept of h. (3)

21.7 For which values of x will: (2)

21.7.1 $\frac{g'(x)}{h(x)}\geq 0$ (3)

21.7.2 $f^{-1}(x-1)<2$ (4)

21.8 Calculate the value(s) of k for which $g(x)=f(x)+k$ has two unequal positive roots. (6)

[29]

QUESTION 22 NW SEP 2019

22.1 Consider the function $f(x)=\left(\frac{5}{6}\right)^x$

22.1.1 Write down the equation of $h,$ the reflection of f in the $y-$axis. (1)

22.1.2 Write down the equation of $f^{-1}(x)$ in the form $y=...$ (2)

22.1.3 For which value(s) of x will $f^{-1}(x)\geq 0$? (3)

22.2 The function defined as $f(x)=ax^2+bx+c$ has the following properties:

- $f'(-2,5)=0$
- $f(1)=0$
- $b^2-4ac>0$
- $f(2,5)=6$

(4)

Draw a neat sketch graph of $f\cdot$ Clearly show all $x-$intercepts and turning point.

[9]

POLYNOMIALS			
	Remainder theorem If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r=f(k)$	$f(x)=(ax+b).Q(x)+R$ $f\left(\frac{-b}{a}\right)=R$	
	Factor theorem	$f\left(\frac{-b}{a}\right)=0$	
	Factorise third degree polynomials A polynomial $f(x)$ has a factor $x-k$ if and only if $f(k)=0$	Factorisation by Inspection Using factor theorem - Long division method - Synthetic method	
CALCULUS			
	Concept	Formula	
	Average gradient	$\frac{f(x+h)-f(x)}{h}$	
	Limit concept	$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$	
	Definition of derivative (first principles)	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	
	Power rule	$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	
	Derivative of a curve Gradient function of a curve	$\frac{dy}{dx}$ $D_x[y]$ y' $f'(x)$	
	Second derivative of f	$f''(x)$	
	Gradient of the tangent to a curve	$m_{\tan}=f'(x)$	
	Equation of tangent line	$y-y_1=f'(x_1).(x-x_1)$	
	$x-$intercept(s) of f	Set $f(x)=0$	
	Turning point of a curve (local maximum or minimum point)	Set $f'(x)=0$	
	Point of inflection	Set $f''(x)=0$	

VOCABULARY			
	First principles	Use definition of derivative	
	Local maximum	The $y-$coordinate of a turning point is higher than all nearby points	
	Local minimum	The $y-$coordinate of a turning point is lower than all nearby points	
	Stationary points	Maximum, minimum or point of inflection	
	Rules of differentiation	Apply power rule	

QUESTION 23 NW SEP 2015 - 2019

23.1 Determine from FIRST PRINCIPLES the derivative of:

23.1.1 $f(x)=3x-x^2$ (5)

23.1.2 $g(x)=\frac{3}{x}$ (5)

23.1.3 $h(x)=x^2-6x$ (5)

23.1.4 $u(x)=2x^2-5x+3$ (5)

23.1.5 $v(x)=-x^2+3x-7$ (6)

23.2 Determine:

23.2.1 $\frac{dy}{dx}$ if $y=(2x)^2-\frac{1}{3x}$ (4)

23.2.2 $\frac{dy}{dx}$ if $y=\frac{2\sqrt{x}-5x^2}{\sqrt{x}}$ (3)

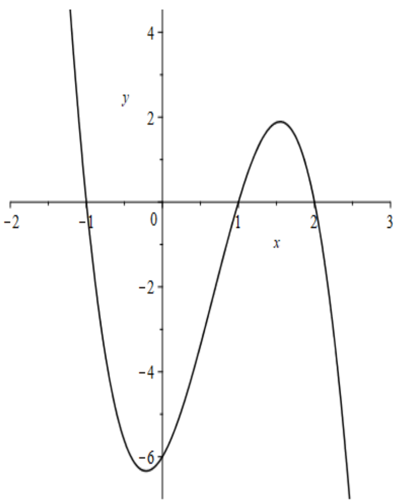
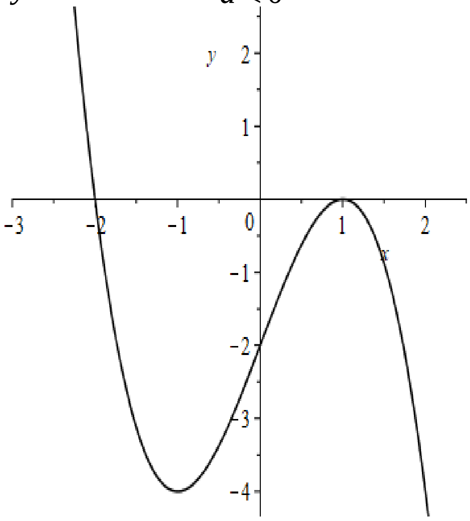
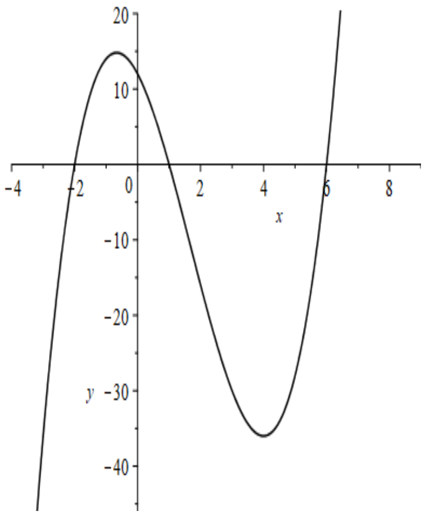
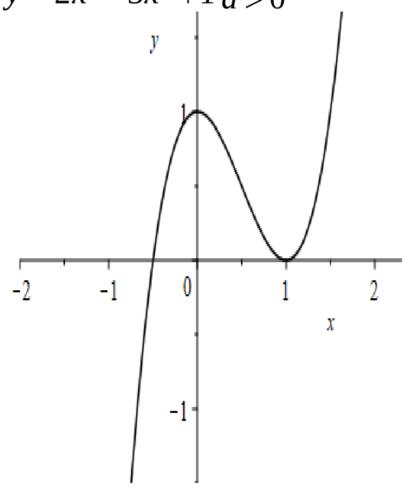
23.2.3 $\frac{dy}{dx}$ if $y=\frac{3x}{5x^2}-\frac{1}{2\sqrt{x}}$ (4)

23.2.4 $\frac{dy}{dx}$ if $y=\frac{2x^2}{3\sqrt{x}}-\frac{2x^3+1}{x^3}$ (5)

23.2.5 $\mathrm{D}_x\left[\pi^3x-\sqrt[3]{x}\right]$ (3)

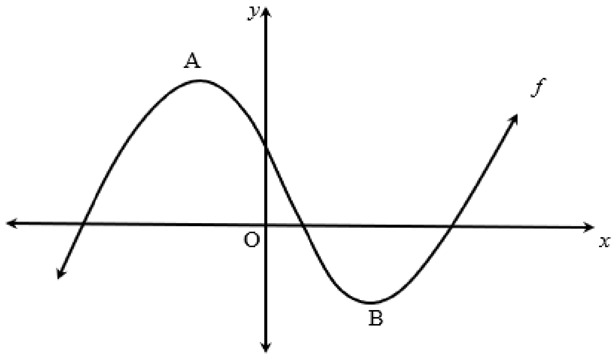
23.2.6 $\mathrm{D}_x\left[\frac{7x^5-3x}{4x}\right]$ (2)

23.2.7 $\mathrm{D}_x\left[15\sqrt[3]{x^4}-\frac{3x^7+x}{4x^3}\right]$ (6)

Orientation of sketch graph	Function	
Concave down-then-Concave up $a<0$ $y=-3x^3+6x^2+3x-6$		$y=-x^3+3x-2$ $a<0$ 
Concave up-then-Concave down $a>0$ $y=x^3-5x^2-8x+12$		$y=2x^3-3x^2+1$ $a>0$ 

QUESTION 24 NW SEP 2015

24.1 The graph of $f(x) = x^3 - 4x^2 - 11x + 30$ is sketched below. A and B are turning points of f



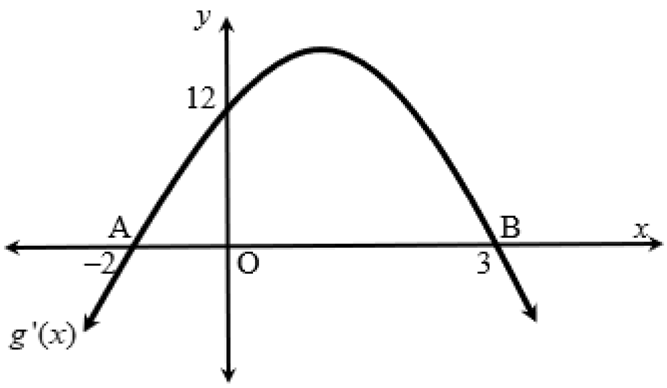
24.1.1 Determine the coordinates of A and B. (6)

24.1.2 Determine the x-coordinate of the point of inflection of f. (2)

24.1.3 Determine the equation of the tangent to f at $x = 2$ in the form $y = mx + c$. (4)

24.1.4 Determine the value(s) of k for which $x^3 - 4x^2 - 11x + 30 - k = 0$ will have only ONE real root. (2)

24.2 The graph of $y = g'(x)$ is sketched below, with x-intercepts at A$(-2; 0)$ and B$(3; 0)$. The y-intercept ofthe sketched graph is $(0; 12)$.



24.2.1 Determine the gradient of g at $x = 0$. (1)

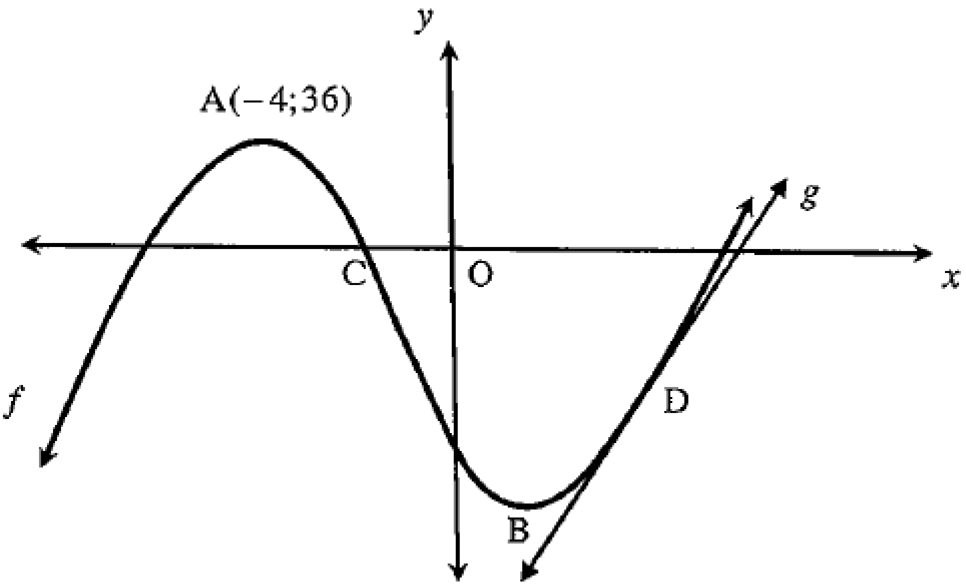
24.2.2 For which value of x will the gradient of g be the same as the gradient in QUESTION 7.2.1? (1)

24.2.3 Draw a sketch graph of $y = g(x)$. Show the x-values of the stationary points and the point of inflection on your sketch. It is not necessary to indicate the intercepts with the axes. (3)

[19]

QUESTION 25 NW SEP 2016

The function defined by $f(x) = x^3 + px^2 + qx - 12$ is sketched below. $\mathrm{A}(-4;36)$ and B are turning points of $f \cdot g$ is a tangent to the graph of f at D.



25.1 Show that $p = 5$ and $q = -8$. (6)

25.2 If $\mathrm{C}(-1;0)$ is an $x-$intercept of f, calculate the other $x-$intercept of f. (4)

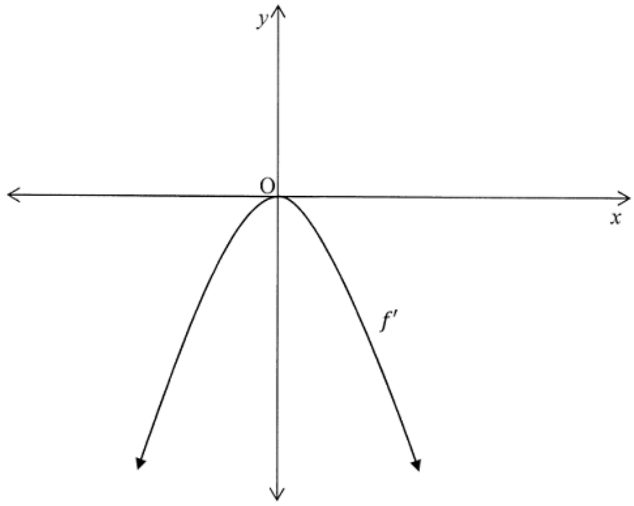
25.3 Determine the equation of g, the tangent to f at $\mathrm{D}(1;-14)$. (4)

25.4 For which values of k will $f(x) = g(x) + k$ have TWO positive roots? (2)

[16]

QUESTION 26 NW SEP 2017

26.1 The sketch below shows the graph of $y = f'(x)$, where f is a cubic function.



26.1.1 Write down the $x-$coordinate(s) of the stationery point(s) of f. (1)

26.1.2 For which value(s) of x is f decreasing? (1)

26.1.3 It is further given that $f(0) = -5$ and $f(-2) = 0$. draw a sketch graph of f in your ANSWER BOOK. (4)

26.2 The graph $h(x) = ax^3 + px$ passes through the point $(3; -2)$. The gradient of the tangent to h at $(0; 0)$ is 3.

26.2.1 Determine the values of a and p. (4)

26.2.2 Determine the gradient of the tangent to h at $x = 3$. (2)

[12]

QUESTION 27 NW SEP 2018

27.1 Given: $f(x) = -2x^3 + 5x^2 + 4x - 3$

27.1.1 Calculate the coordinates of the $x-$intercepts of f if $f(3) = 0$. Show all calculations. (4)

27.1.2 Calculate the $x-$values of the stationary points of f. (4)

27.1.3 For which values of x is f concave up? (2)

27.2 The function g, defined by $g(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $g(-2) = g(4) = 0$
- The graph of $g'(x)$ is concave up.
- The graph of $g'(x)$ has $x-$intercepts at $x = 0$ and $x = 4$ and a turning point at $x = 2$.

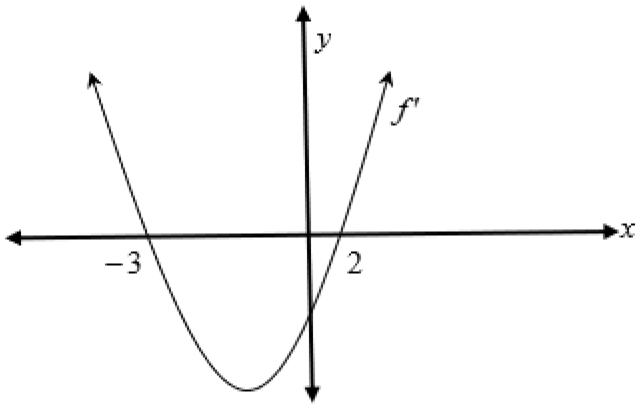
27.2.1 Use this information to draw a neat sketch graph of g without actually solving for $a,\ b,\ c$ and d. Clearly show all $x-$intercepts, $x-$values of the turning points and $x-$value of the point of inflection on your sketch. (4)

27.2.2 For which values of x will $g(x).g''(x) > 0$? (3)

[17]

QUESTION 28 NW SEP 2019

The graph of $f'(x) = x^2 + bx + c,$ where f is a cubic function, is sketched below. The derivative function f' cuts the $x-$axis at $x = -3$ and $x = 2.$



28.1 For which values of x is graph f increasing? (2)

28.2 At which value of x does graph f have a local maximum value? (1)

28.3 Determine the equation of $f'(x)$. (2)

28.4 If $f(x) = px^3 + qx^2 + rx + 10,$ show that $p = \frac{1}{3},\ q = \frac{1}{2}$ and $r = -6.$ (4)

28.5 For which value(s) of x is graph f concave down? (3)

[12]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$;

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$area \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

