Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sport Ontwikkeling
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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

MID-YEAR EXAMINATION 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 2 answer sheets, 7 diagram sheets and an information sheet of 2 pages.

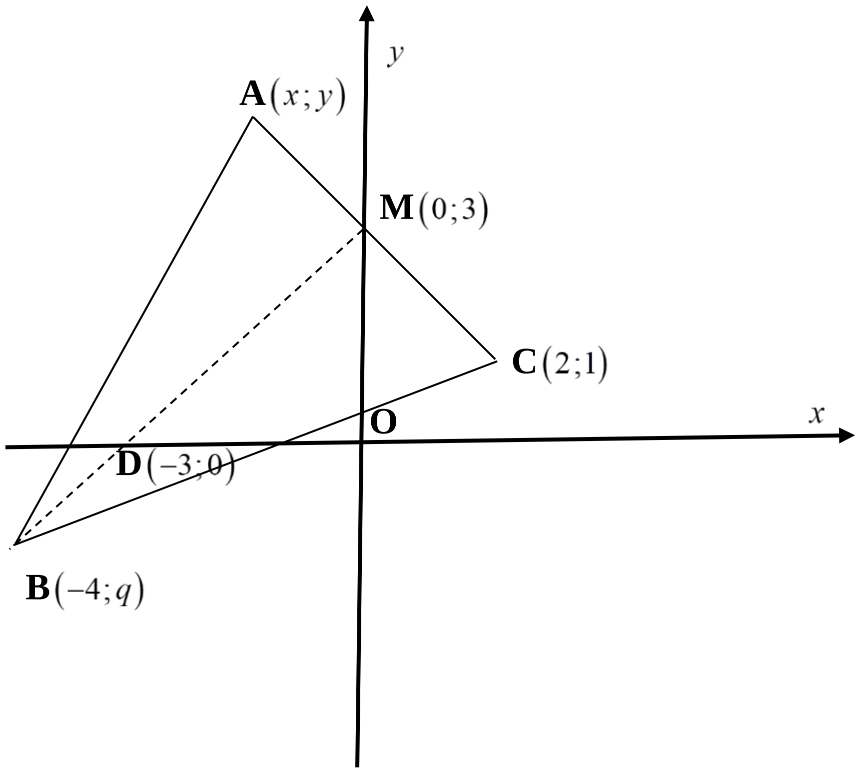
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

In the diagram below ΔABC has the vertices $A(x;y)$, $B(-4;q)$ and $C(2;1)$ in the Cartesian plane. $D(-3;0)$ and $M(0;3)$ are given. M is the midpoint of the line segment AC and BDM is a straight line.

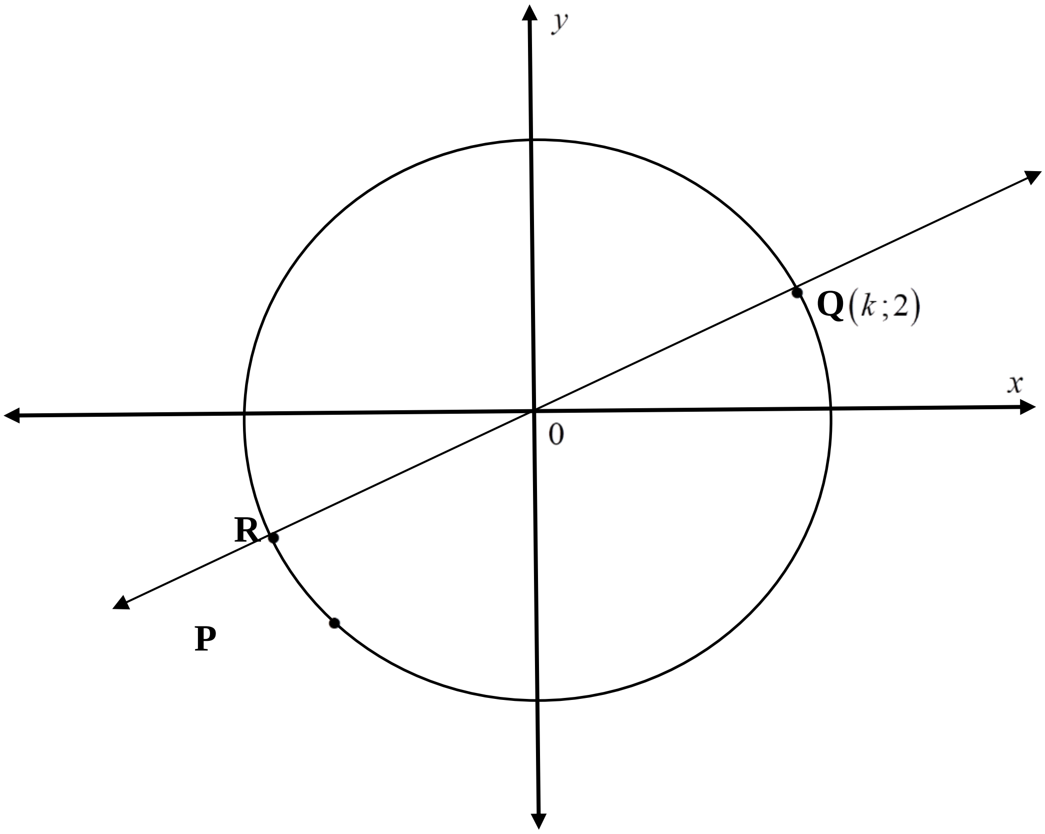


1.1	Determine the gradient of AC.	(2)
1.2	Determine the coordinates of A.	(3)
1.3	Determine the equation of the line passing through B, D and M in the form $y = mx + c$.	(2)
1.4	Show that $B\hat{M}C = 90°$.	(1)
1.5	Determine the size of $M\hat{D}O$.	(2)
1.6	Determine the length of BC, leaving your answer in simplified surd form.	(4)

[14]

QUESTION 2

2.1 In the diagram below, a circle centred at the origin is drawn. Points $P(-4;-3)$ and $Q(k;2)$ lie on the circumference of the circle. A straight line $y = ax + q$ passes through point Q ,the origin and produced to cut the circle at R.



2.1.1 Determine the equation of the circle. (2)

2.1.2 Calculate the value of k. (2)

2.1.3 Determine

(a) the equation of RQ (2)

(b) the coordinates of R. (2)

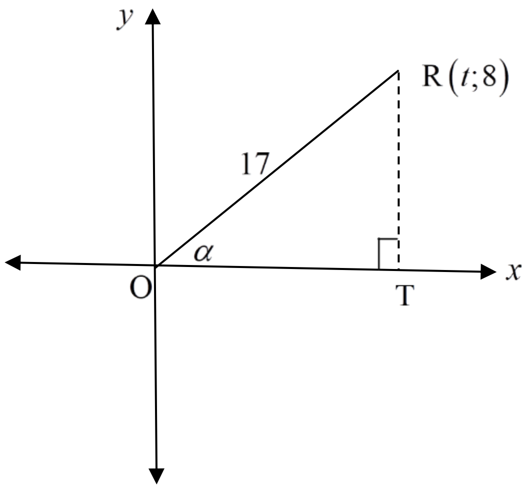
2.1.4 If a tangent is drawn to the circle at point P, determine the equation of the tangent. (4)

2.2 On the grid provided, plot the graph of $\frac{x^2}{25} + \frac{y^2}{16} = 1$. (3)

[15]

QUESTION 3

The point $R(t;8)$ lies in the first quadrant such that $OR = 17$ units and $T\hat{O}R = \alpha$ as shown in the diagram below:



Determine the value of

3.1 t (2)

3.2 $\cos(180° + \alpha)$ (2)

3.3 α (2)

[6]

QUESTION 4

4.1 Simplify the following expression

$$\frac{\sin(-\theta)\cos(\theta+180°)-\cos(90°+\theta)}{\sin(540°-\theta)}$$ (6)

4.2 Hence without using a calculator determine the value of

$$\frac{\sin(-\theta)\cos(\theta+180°)-\cos(90°+\theta)}{\sin(540°-\theta)} \text{ if } \theta = 60°.$$ (2)

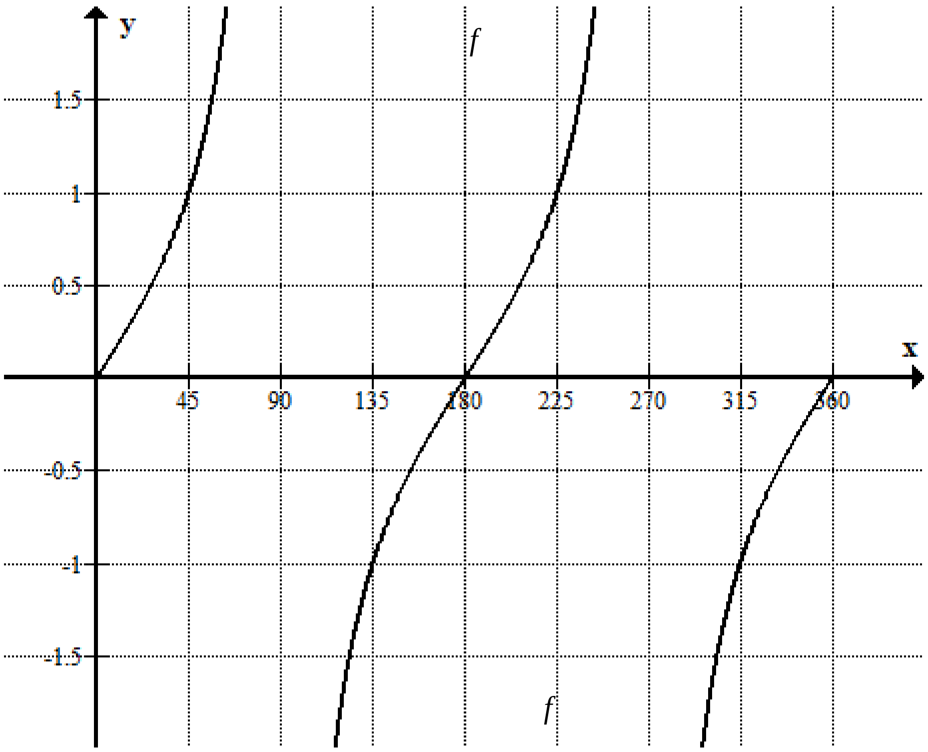
4.3 Prove the following identity: $\frac{1}{\cos^2\theta} - \tan^2\theta = 1.$ (3)

4.4 Simplify without using a calculator: $\frac{2\cos 20° \sin 120° \sin 200°}{\sin 110° \sin 340°}$ (5)

[16]

QUESTION 5

In the diagram below, the graph of $f(x) = a\tan x$ is drawn for the interval $x \in [0°;360°]$.



5.1 On the same system of axes in the diagram sheet, sketch the graph of g, where $g(x) = \sin 2x$, for the interval $x \in [0°;360°]$. (3)

5.2 Determine the value of a. (1)

5.3 Write down the periods of

(a) f and (1)

(b) g (1)

5.4 What is the domain of f? (2)

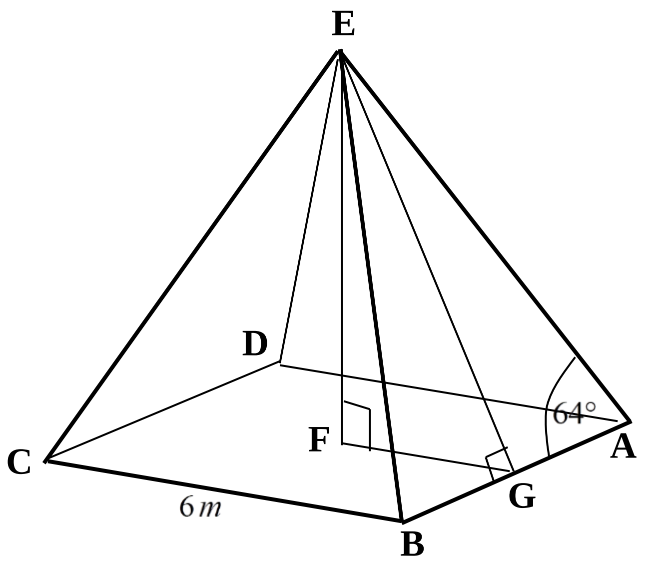
5.5 Find the values of x, for which $f(x) = g(x)$, where $0° < x < 180°$. (2)

5.6 For which values of x, $x \in (180°;360°)$, is $f(x) < g(x)$? (2)

[12]

QUESTION 6

The diagram shows a pyramid shaped 'The Louvre in Paris, France'.
Each face is an isosceles triangle with base angles of $64°$.
The base is a square of side $6\ m$. EG is the slant height of the pyramid.
EF is the perpendicular height of the pyramid.



The Louvre in Paris, France

The formulae below can be used to answer the questions that follow:

$$\text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Volume of pyramid} = \frac{1}{3} \times (\text{area of base}) \times (\text{height})$$

$$\text{Surface Area} = 4\left(\frac{1}{2} \times \text{base} \times \text{slant height}\right)$$

6.1 $\quad A\hat{E}G = ...$ (1)

6.2 Determine the length of edge AE. (3)

6.3 Calculate the height EF. (4)

6.4 Determine the volume of the pyramid. (2)

6.5 The pyramid is to be wrapped in single layer of foil, with no overlaps.

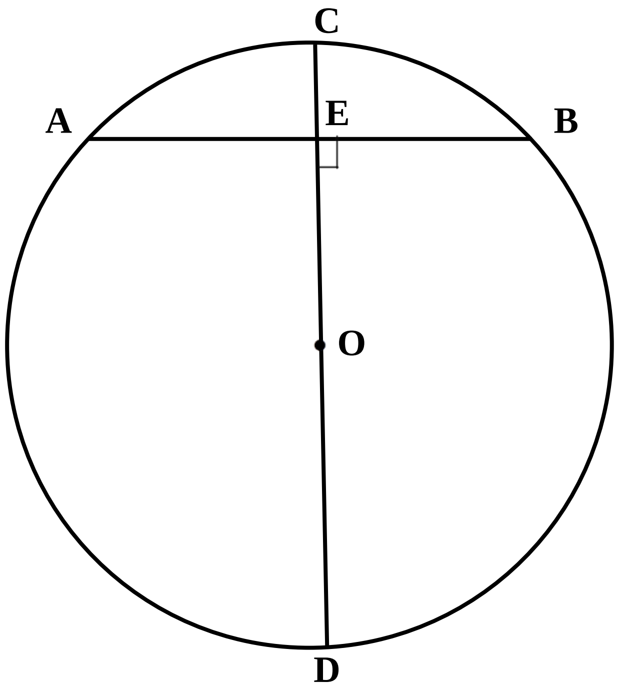
Calculate the total area of foil that would be needed. (3)

[13]

QUESTION 7

In the diagram below, O is the centre.
AB is perpendicular to CD.

$AB = 6$ units and $OE = 4$ units.



Determine giving reasons the lengths of the following sides:

7.1 OB (3)

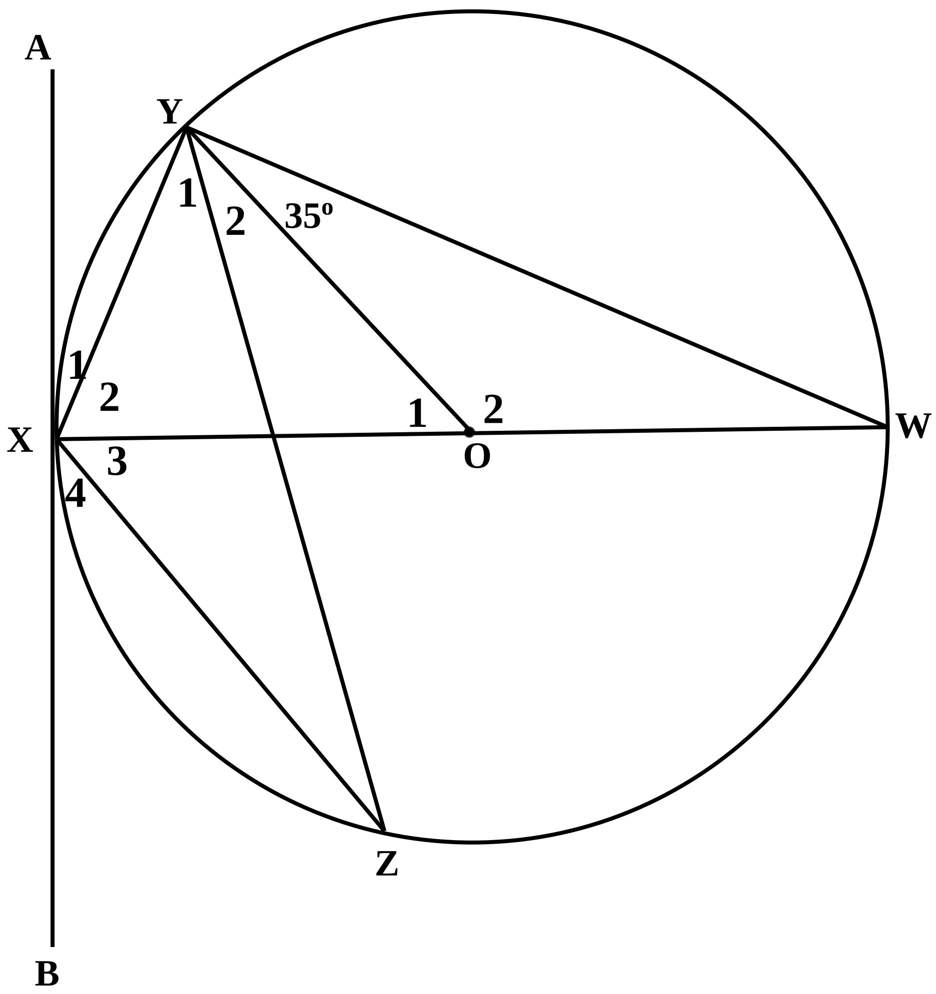
7.2 BD (3)

[6]

QUESTION 8

In the diagram below, AB is a tangent to the circle (with centre O) at X.

W, X, Y and Z are points on the circle. $O\hat{Y}W = 35°$.



8.1 Calculate (with reasons) the size of $X\hat{Z}Y$. (3)

8.2 If $\hat{X}_4 = 30°$, calculate (with reasons) the size of $\hat{Y}_2$. (3)

8.3 Hence calculate (with reasons) the size of $\hat{X}_3$. (2)

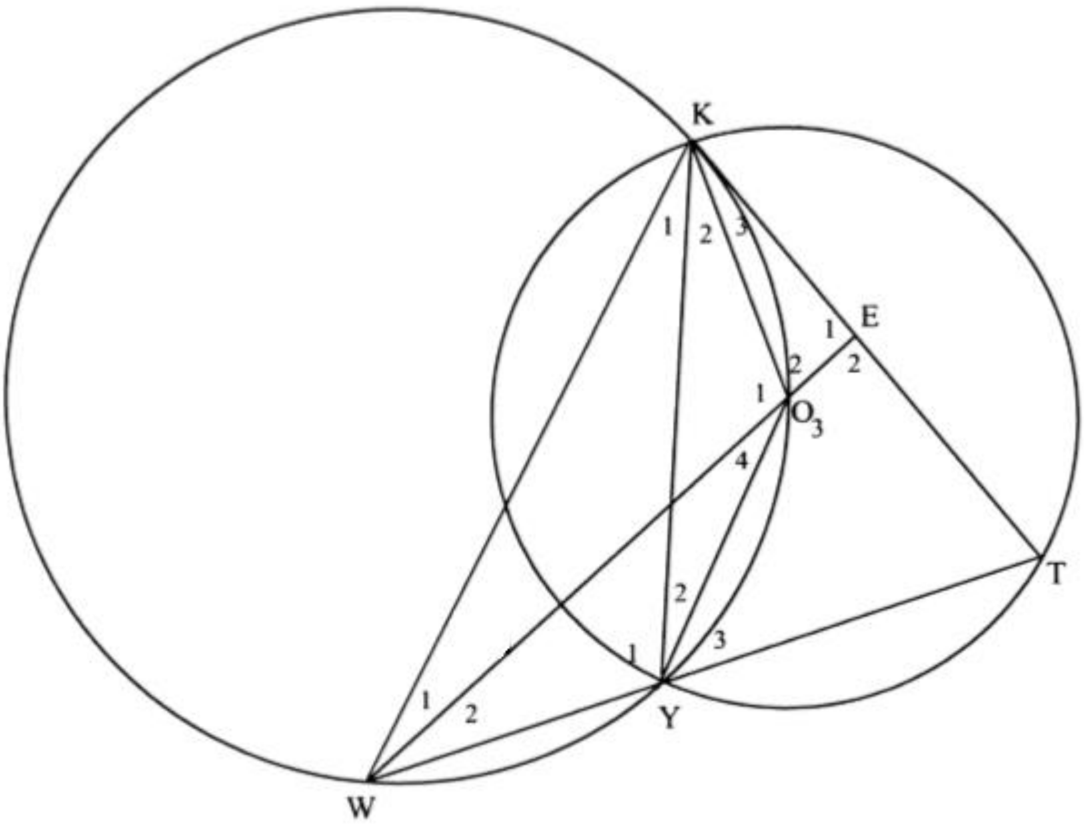
8.4 If $XY = 8$ units and $OY = 8,5$ units determine the length of YW. (3)

[11]

QUESTION 9

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W. WO produced meets KT at E.

$\hat{K}_3 = 20°$.



9.1 Determine giving reasons FOUR other angles, each equal to $20°$. (8)

9.2 Determine the size of $K\hat{O}Y$. (2)

9.3 Determine the size of $\hat{T}$. (2)

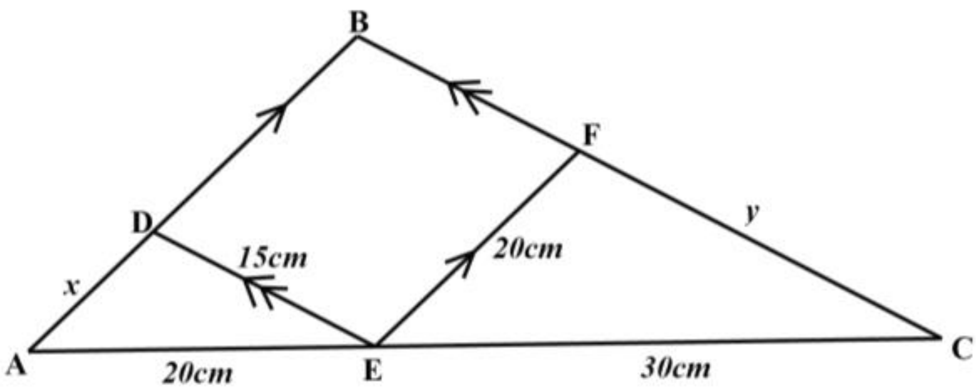
9.4 Determine the size of $\hat{Y}_3$. (5)

[17]

QUESTION 10

Given ΔABC with D a point on AB such that DE // BC and F is a point on BC such that AB // EF.

Given $DE = 15\,cm$, $EF = 20\,cm$, $EC = 30\,cm$ and $AE = 20\,cm$.



Determine, giving reasons, the values of x and y. (6)

[6]

QUESTION 11

A certain model of an electric fan has a rotation rate of 3 700 revolutions per minute and blades of $0,25\,m$ in radius.

11.1 Determine the angular velocity in radians per seconds. (4)

11.2 Hence, calculate the speed at the tip(circumferential) of the blade. (3)

[7]

QUESTION 12

A bicycle wheel has a diameter of $0,72\,m$ and the bicycle is moving at $6\,m.s^{-1}$.

12.1 What is the peripheral velocity of the wheel? (1)

12.2 Calculate the angular velocity. (3)

12.3 The arc length of a sector of the wheel as shown in the diagram below is $\frac{1}{3}$ of the circumference of the wheel.

12.3.1 Determine the angular position θ in radians. (4)

12.3.2 Calculate the area of the sector. (3)

12.4 A chord with a length of $50\,cm$ divides the bicycle wheel into two segments. Calculate the heights of the segments. (5)

[16]

QUESTION 13

The Gautrain takes 35 minutes to travel from Pretoria to Johannesburg at an average speed of $140\,km/h$.

Gautrain

13.1 Determine the distance it travels from Pretoria to Johannesburg. (3)

13.2 When leaving Johannesburg Station for O. R. Tambo Airport the train covers an arc length (s) $8\,km$ of a circumference with a radius (r) $9\,km$ for 5 minutes.

13.2.1 Determine the angular displacement of the train. (2)

13.2.2 Determine the circumferential velocity of the train along this arc length. (4)

13.3 It costs R 41 to travel from Pretoria to Johannesburg by Gautrain.

Calculate the total costs of travel from Pretoria to Johannesburg and back in 20 days of a month? (2)

[11]

TOTAL: [150]

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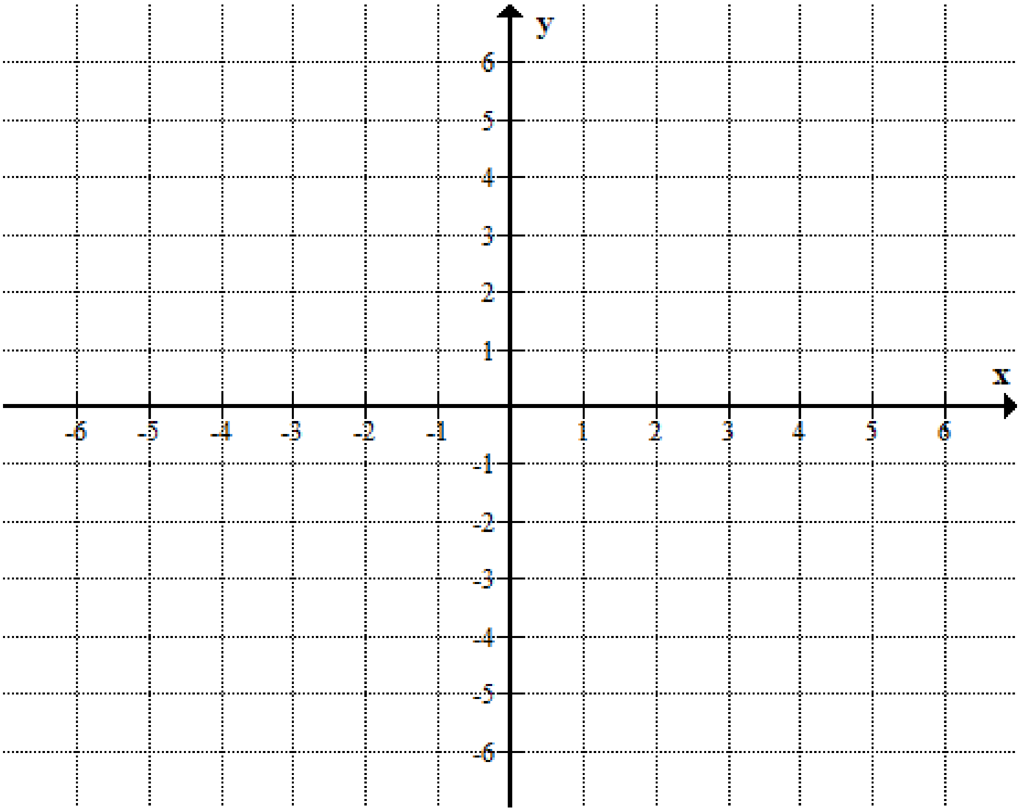
QUESTION 2.2



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QUESTION 5

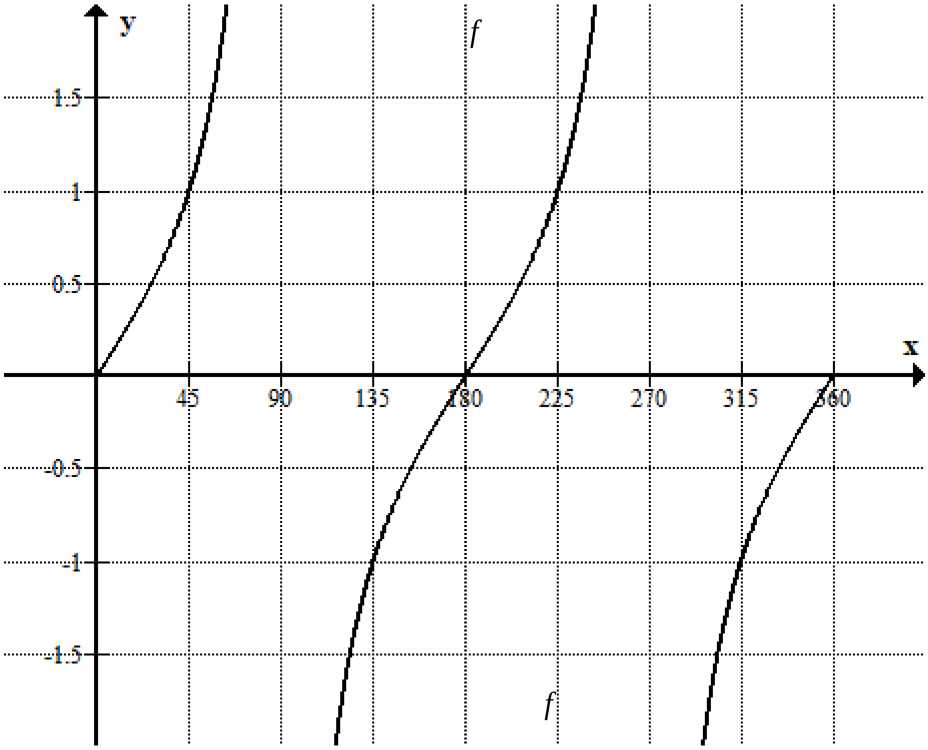

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DIAGRAM SHEET 1

QUESTION 1

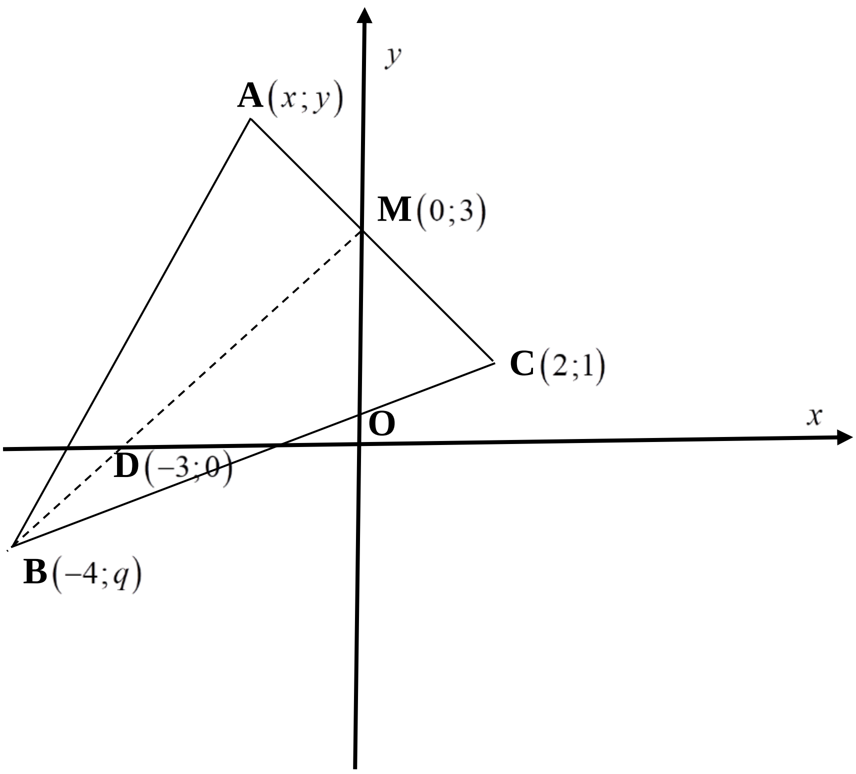

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DIAGRAM SHEET 2

QUESTION 2

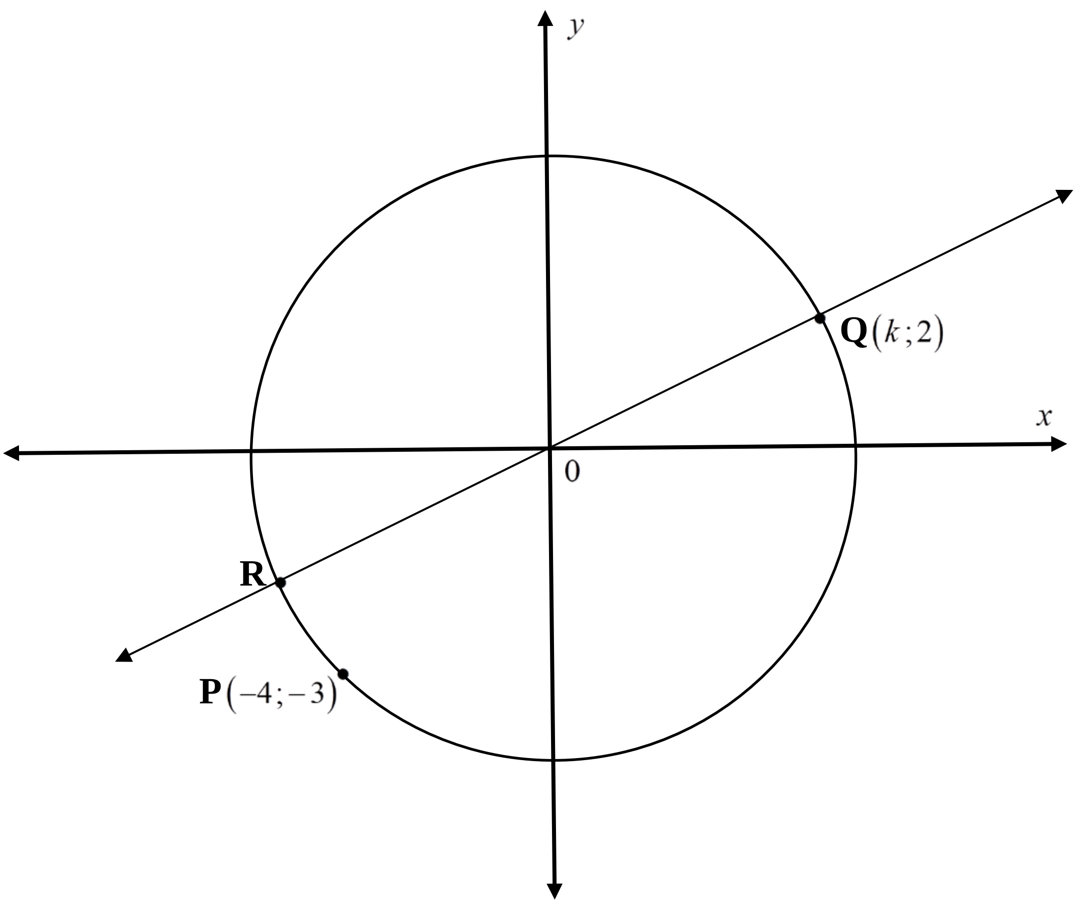

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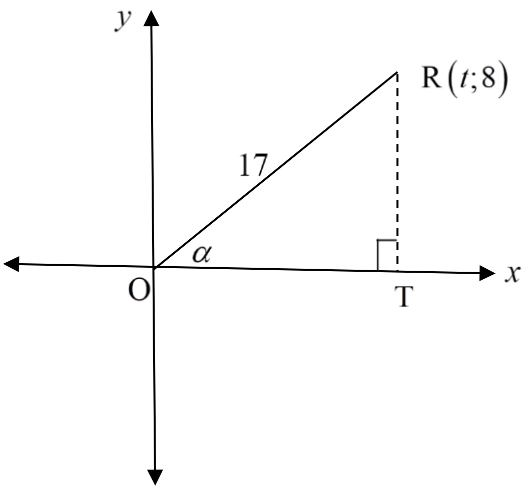
DIAGRAM SHEET 3

QUESTION 3



QUESTION 6

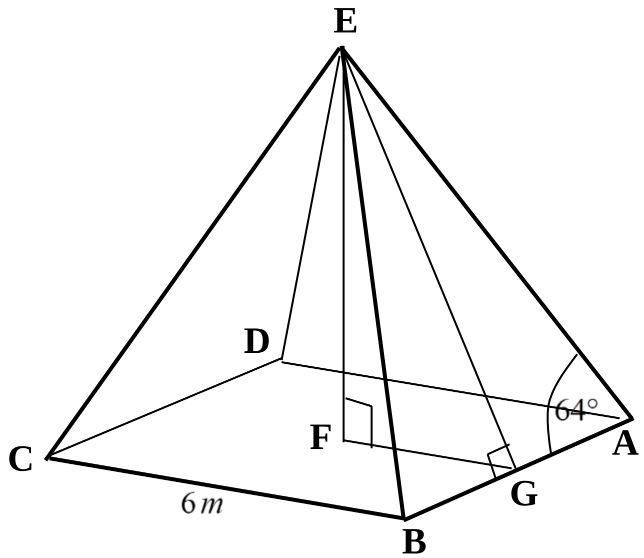


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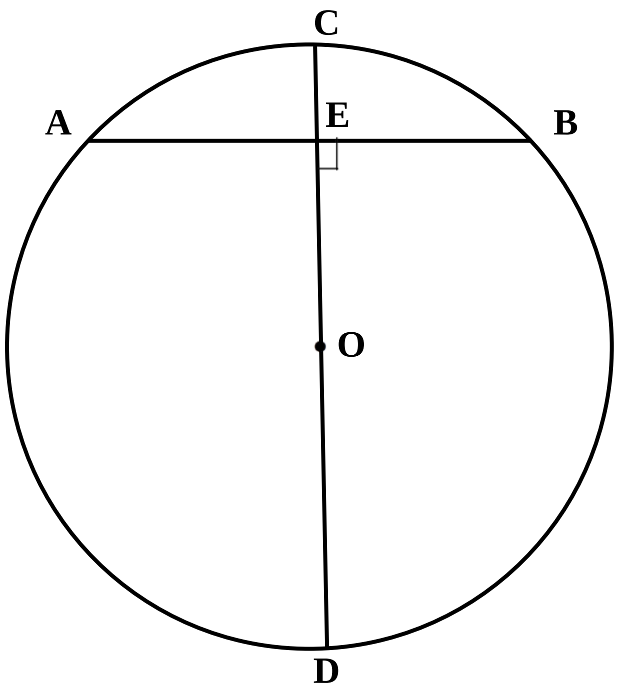
DIAGRAM SHEET 4

QUESTION 7



NAME OF LEARNER: ………………………………………………. CLASS: ……………

DIAGRAM SHEET 5

QUESTION 8

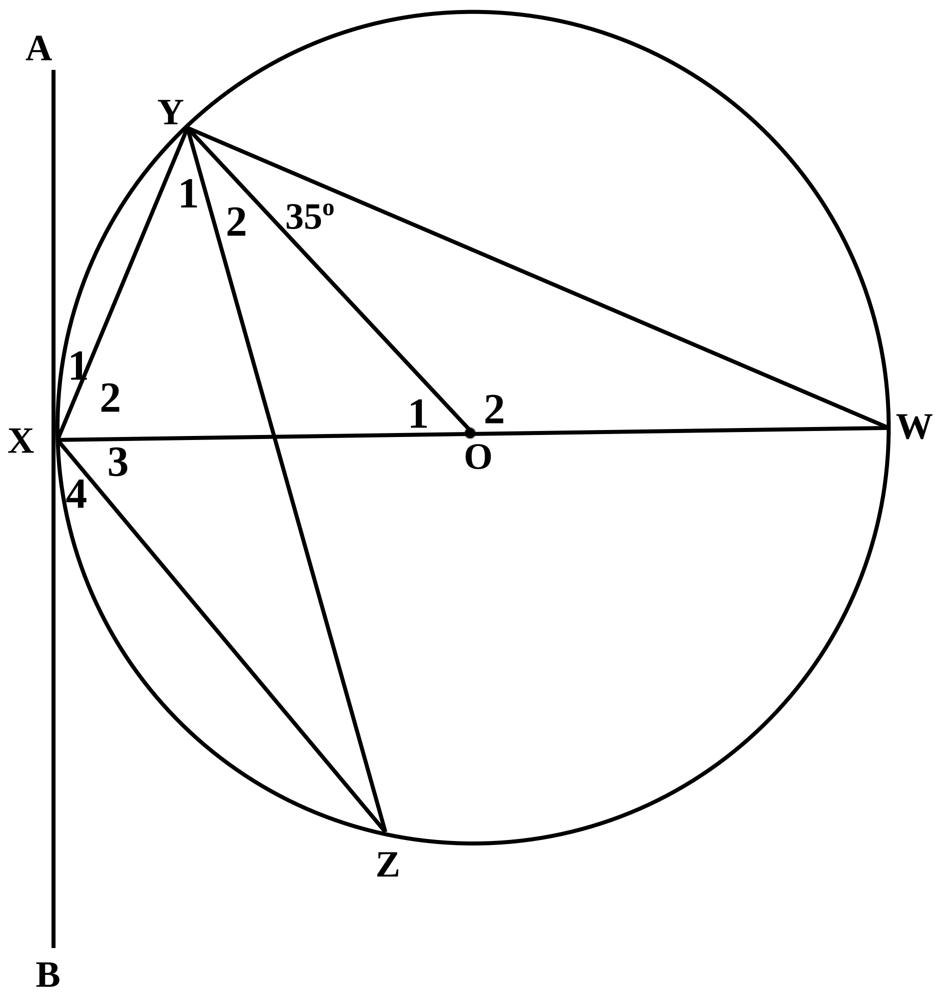


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DIAGRAM SHEET 6

QUESTION 9

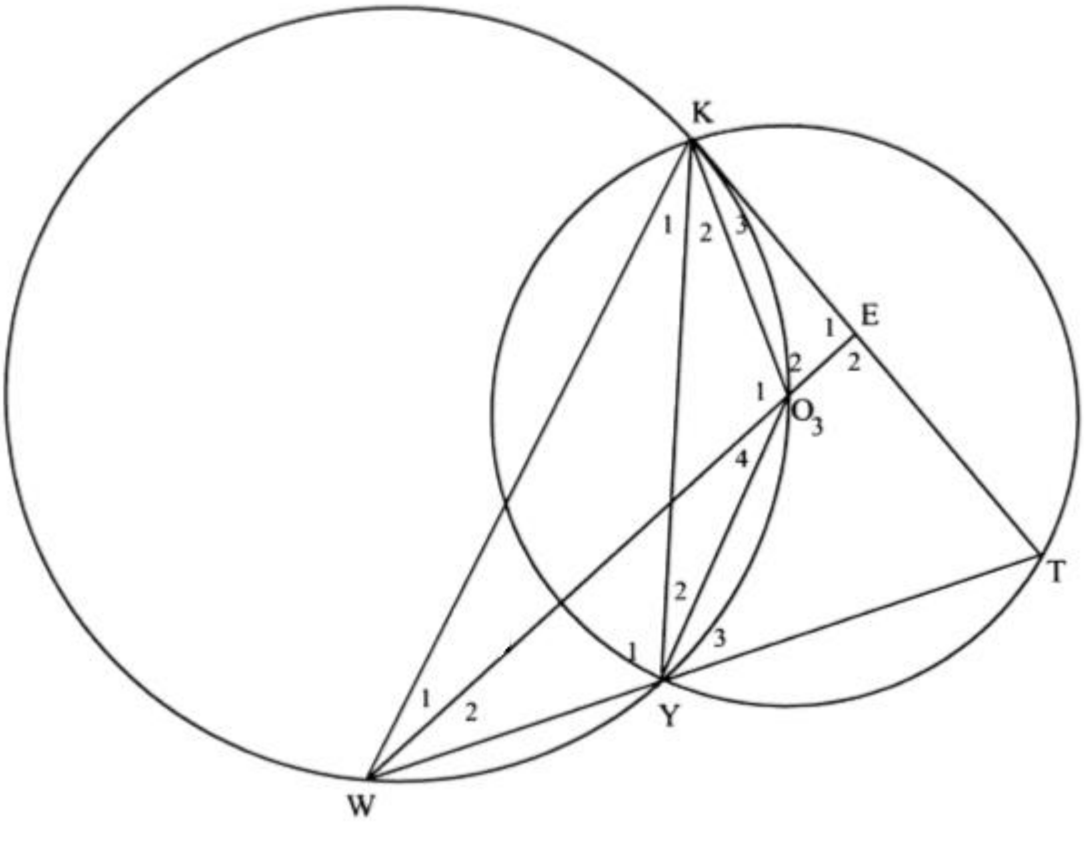

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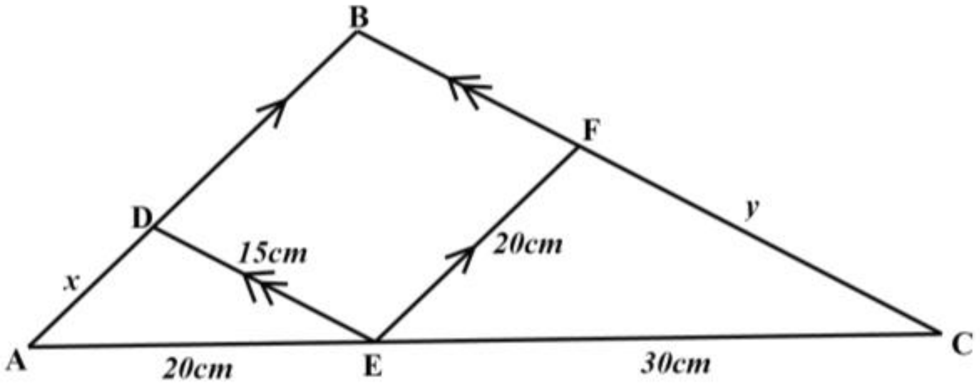
DIAGRAM SHEET 7

QUESTION 10



INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\frac{b}{2a}$ $\qquad y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b$, $\quad a > 0$, $a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 - i)^n$ $\qquad A = P(1 + i)^n$

$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$

$f^/(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\, of\ \Delta \mathrm{ABC} = \frac{1}{2} ab \cdot \sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \mathrm{cos}ec^2\theta$

$f^/(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$

$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360^0 n$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, o_i = i^{th} ordinate and n = number of ordinates

OR

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates