Education and Sport Development

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NORTH WEST PROVINCE

GRADE 12

MATHEMATICS PAPER 1

MID YEAR EXAMINATION 2019

MARKS: 150

TIME: 3 Hours

This question paper consists of 8 pages (formula sheet included)

NW/JUNE/MATH/ EMIS/6*******

INSTRUCTIOS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **09** questions
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers
4. Answers only will not necessary be awarded full marks.
5. You may use an approved scientific calculator (non – programmable and non graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessary drawn to scale
8. An information sheet , with formulae, is included at the end of the question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x

1.1.1 $x^2 - x = 0$ (2)

1.1.2 $4 - 10x - 3x^2 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{11 - x} = 1 + x$ (5)

1.1.4 $-(x-4)(x+3) < 0$ (3)

1.2 Solve for x and y in the following set of simultaneous equations:

$x^2 + xy + y^2 = 7$ and $4^{x+2}.8^{y+1} = 2^{1-x}$ (7)

1.3 Show that the equation $x^2 - x = p^2$ where $x,\ p \in \Re$, will never have equal roots for any value of p. (5)

[26]

QUESTION 2

A quadratic pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$

2.1 Determine the value of a. (3)

2.2 Determine the formula for T_n. (3)

2.3 Hence, prove that the sum of any two consecutive numbers in the sequence is exactly divisible by 2. (3)

[09]

QUESTION 3

3.1 To drill a borehole costs R700 for the first metre, and then the cost per metre rises by R450 for every metre drilled, up to a depth of 50 metres. Hereafter the cost rises by R700 per metre.

3.1.1 How much will it cost to drill 50 metres? (2)

3.1.2 Determine the depth of the borehole if R85 050 is to be spent. (4)

3.2 The sum of the first n terms of an arithmetic series is given by $S_n = 2n - n^2$, Calculate the seventh term of the series. (3)

3.3 Calculate $\sum_{m=0}^{9} \frac{(-1)^{m+1}}{2^m}$ (correct to TWO decimal places) (5)

3.4 If $(1-x)$, $(1+x)$ and $(2x+7)$ are the first three terms of an infinite converging geometric sequence, calculate

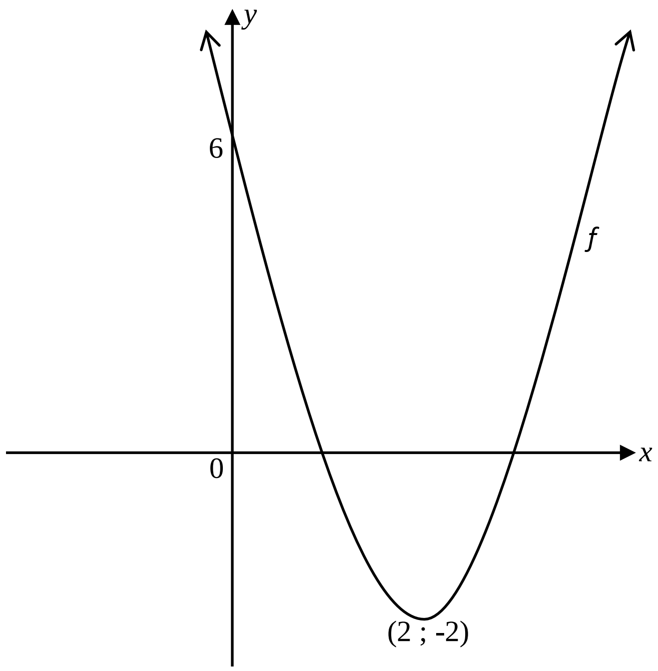
3.4.1 the value(s) of x. (7)

3.4.2 the sum to infinity of the series. (2)

[23]

QUESTION 4

4.1 The figure below shows the graph of f. The graph intersects the y-axis at $y=6$ and the turning point has the coordinates $(2\,;-2)$.



4.1.1 Determine the equation of f in the form $f(x)=ax^2+bx+c$ (4)

4.1.2 Calculate the x – intercepts of f. (4)

4.1.3 Describe the transformation of f to g if $g(x)=2x^2$. (2)

4.1.4 Restrict the domain of g such that its inverse is a function. (2)

4.2 Given $h(x)=\log_2 x$.

4.2.1 Determine h^{-1} in the form $y=$...... (2)

4.2.2 Sketch the graphs of h and h^{-1} on the same set of axes.
Indicate the intercepts. (4)

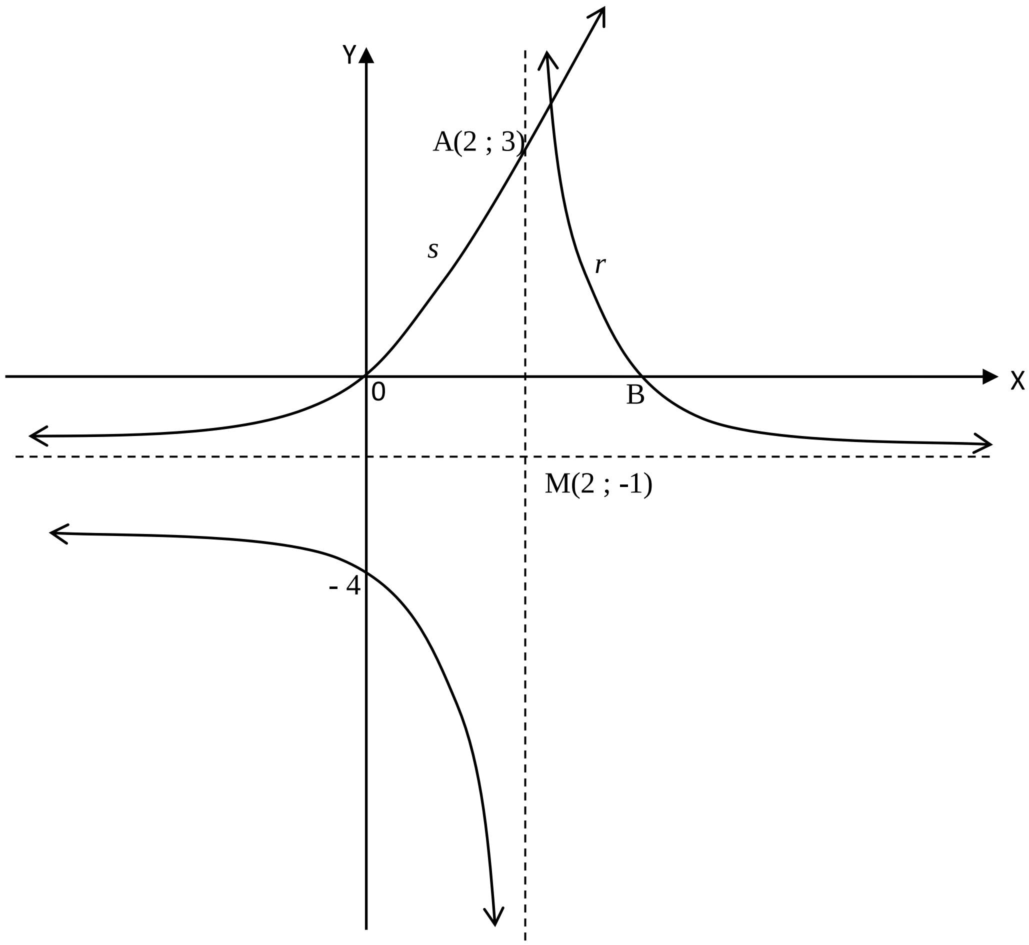
4.2.3 If $(2;a)$ lies on h, calculate the value of a. (2)

4.2.4 Solve for x if $\log_2 x \le 1$. (2)

[22]

QUESTION 5

Sketched are the graphs of $r(x)=\dfrac{k}{x+p}+q$ and $s(x)=a^{x}+b$. The graph of s passes through the origin and r intersects the $y-$axis at -4. M(2 ; -1) is the intersection of the asymptotes of r. A(2 ; 3) lies on s.



5.1 Determine the equations of r and s (5)

5.2 Calculate the coordinates of B, the $x-$intercept of r. (3)

5.3 Determine the equation of the line of symmetry of r where $m<0$. (2)

5.4 Calculate the coordinates of W, if W is the reflection of A on the line of symmetry in 5.3 above. (2)

5.5 For which values of x will $s(x).r(x)<0$? (3)

5.6 What is the range of $k(x)=s(x)-r(x)$? (3)

[18]

QUESTION 6

6.1 James bought a new expensive German Sports car. Calculate how many years it will take for the value of a car to decrease to 50% of its original value if depreciation is calculated at 10% per annum using straight line method. (4)

6.2 Mpho invests R42 000 for 5 years. She receives interest of r % per annum compounded monthly. The money grew to R75 000 at the end of the 5 year period. Calculate the interest rate Mpho received. (4)

6.3 Lebo opens a savings account for 5 years at an interest rate of 7% per annum compounded quarterly. Determine the effective interest rate. (3)

[11]

QUESTION 7

7.1 Given: $f(x)=\frac{1}{2}x^2$.

7.1.1 Determine $f'(x)$ from the **FIRST PRINCIPLES**. (5)

7.1.2 Calculate the average gradient of f between $x=-5$ and $x=3$. (4)

7.2 Evaluate: $D_x\left[2x^3-4x-\pi\right]$ (2)

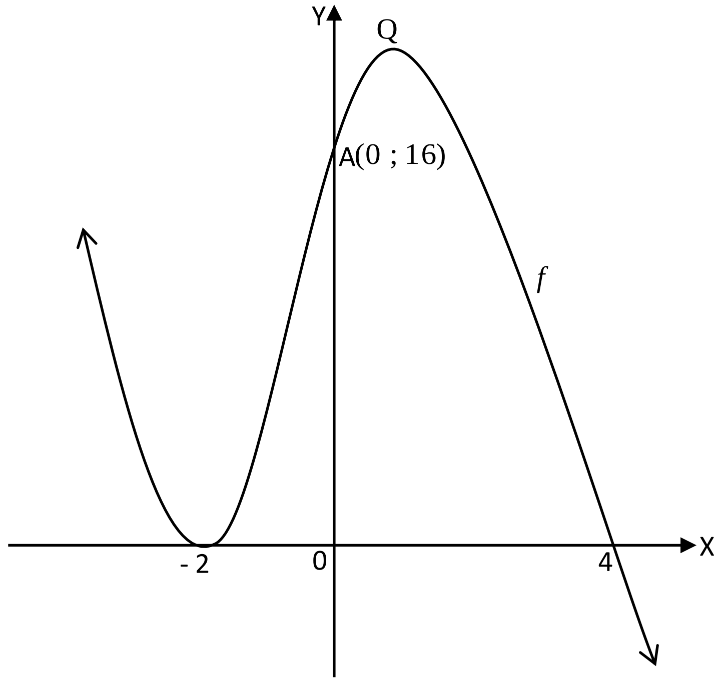
7.3 Determine $\frac{dy}{dx}$ if $y=\sqrt{x}\left(x-\frac{1}{x}\right)$ (4)

7.4 Determine the x – coordinate of the point on the curve $y=-x^2-4x+12$ where the gradient of the tangent is -14. (3)

[18]

QUESTION 8

The graph below represents $f(x) = ax^3 + bx + d$. The function has a root of 4 and a local minimum value on the x – axis at $x = -2$. The graph of f has a y intercept at A(0 ; 16).



8.1 Show that the equation of f is given by $f(x) = -x^3 + 12x + 16$. (4)

8.2 Calculate the coordinates of Q, the turning points of f. (5)

8.3 Determine the coordinates of the point of inflection of t if $t(x) = -f(x)$. (4)

8.4 Determine the value(s) of x for which $\frac{f'(x)}{f(x)} \leq 0$ (3)

[16]

QUESTION 9

The events A and B are independent. P (A) = 0,4 and P (B) = 0,5

Determine:

9.1 P (A and B) (2)

9.2 P (A or B) (2)

9.3 P (not A and not B) (3)

[07]

TOTAL MARKS: 150

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$