Education and Sport Development

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 1

MID YEAR EXAMINATION 2018

MARKS: 150

TIME: 3Hours

This question paper consists of 8pages (Graph and formula sheets included)

INSTRUCTIOS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers
4. Answers only will not necessary be awarded full marks.
5. You may use an approved scientific calculator (non – programmable and non graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessary drawn to scale
8. Number of diagram sheets, for answering
9. An information sheet , with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x

1.1.1 $x^2 - 5x = -6$ (3)

1.1.2 $3x^2 - 4x - 2 = 0$ Correct to TWO decimal places (4)

1.1.3 $(3x+1)(x-4) > 0$ (3)

1.2 Solve for x and y:

$$\frac{4^x}{2^y} = 256$$
$$x^2 - xy + y^2 = 19$$ (8)

1.3 If $x^2 = 3$, calculate the value(s) of x^5. (3)

[21]

QUESTION 2

The sequence: -67 ; x ; y ; -28 ; -19 ; ... has a quadratic pattern.

2.1 Determine the value of x and y. (6)

2.2 Find an expression for the nth term. (4)

2.3 Prove that the sequence of numbers will never contain a positive term. (3)

[13]

QUESTION 3

3.1 The fourth term of a geometric progression is 24 and the ninth term is 768.

Determine the first three terms of the progression. (5)

3.2 If $S_n = 2n^2 + 3n$ then

3.2.1 Calculate T_{12} (3)

3.2.2 Find T_n in its simplest form. (4)

3.3 Calculate the value of $\sum_{n=2}^{18}(2n - 1)$. (3)

3.4 In a converging geometric series $S_\infty = \frac{40}{3}$ and $T_2 = \frac{5}{2}$; calculate the possible value(s) the first term of the series. (7)

[22]

QUESTION 4

Given: $f(x) = \dfrac{x+3}{x+1}$

4.1 Calculate the *x*and *y*intercepts of *f*. (3)

4.2 Show that*f* can be written as $f(x) = \dfrac{2}{x+1} + 1$. (2)

4.3 Write down the equation of the

4.3.1 vertical asymptote of *f*. (1)

4.3.2 horizontal asymptote of *f*. (1)

4.4 Draw a sketch graph of *f* showing clearly the intercepts and asymptotes. (4)

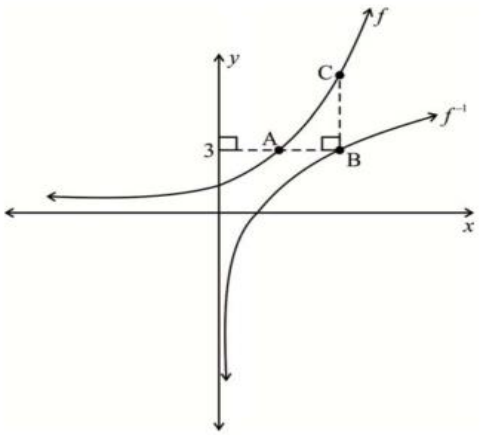
4.5 Use your graph in 5.4 to solve for *x*, $f(x) \geq 3$ (2)

4.6 Determine $\sum_{0}^{3} f(x)$. (3)

[16]

QUESTION 5

5.1 In the diagram below $f(x) = 2^x$.



5.1.1 Write down the equation of f^{-1} in the form $y = ...$ (2)

5.1.2 Calculate the length of AB. (5)

5.1.3 Determine the average gradient of f between $x = 1{,}58$ and $x = 8$. (3)

5.1.4 Write down the domain of f^{-1}. (1)

5.2 Draw the graph of a parabola given by $h(x) = ax^2 + bx + c$ under the conditions $a < 0$; $b < 0$; $c > 0$ and $\Delta > 0$. [5]

[16]

QUESTION 6

6.1 Mpho bought a house for R650 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 15 years. Interest is charged at 11% p.a. compounded monthly.

6.1.1 Calculate the annual effective interest rate of the loan. (3)

6.1.2 Calculate Mpho's monthly instalments. (4)

6.1.3 Calculate the outstanding balance of the loan after 8 years. (4)

6.2 Ben's investment grew to R50 000. The interest rate charged at 18% per annum compounded monthly. If his monthly deposits were R2 300, how many equal payments did Ben make? (5)

[16]

QUESTION 7

7.1 Given: $f(x) = x^2 - 6x$

7.1.1 Determine $f'(x)$ from the **FIRST** principles. (5)

7.1.2 A tangent to the graph of f has gradient -4 and x-intercept $(a\ ;\ 0)$.

Determine the value of a. (6)

7.2 Evaluate: $D_x\left[\sqrt[3]{x} + x^2 + 4x\right]$ (3)

7.3 Determine $\frac{dy}{dx}$ if:

7.3.1 $xy = 5$ (2)

7.3.2 $y = \frac{2x^3 - x}{\sqrt{x}}$ (5)

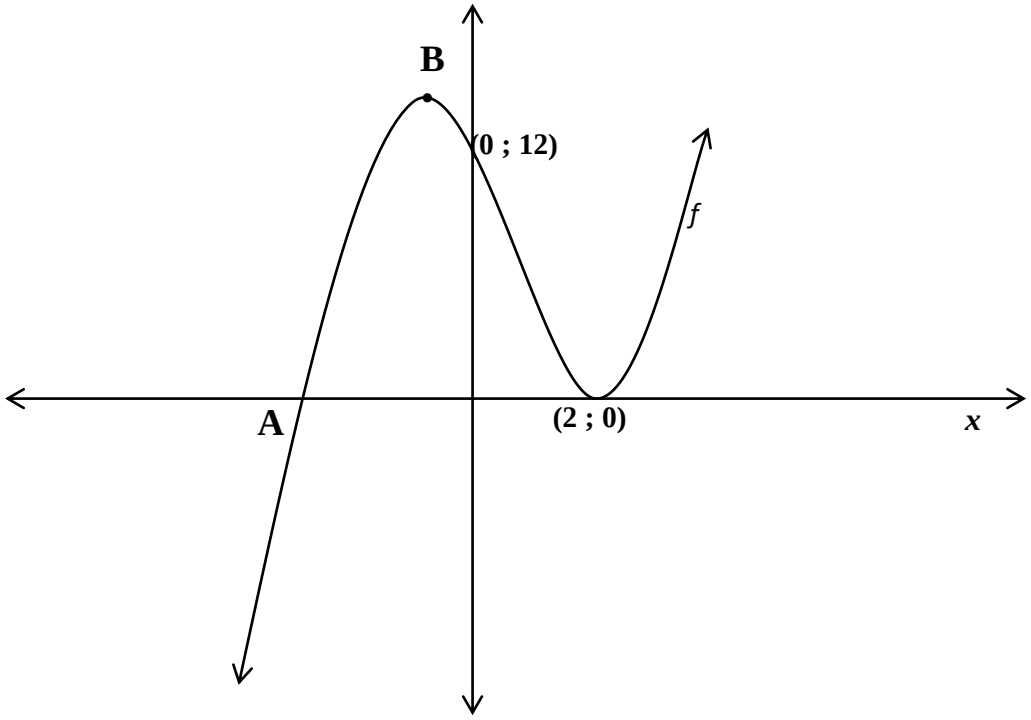
[21]

QUESTION 8

8 The sketch below shows the curve of $f(x) = x^3 - x^2 - 8x + 12$.

The curve has a y - intercept at (0;12) and turning points at (2;0) and B.

The point A is an x-intercept.



8.1 Calculate the coordinates of A. (5)

8.2 Calculate the x- coordinate of B. (4)

8.3 Write the values of x for which $f'(x) > 0$ (3)

8.4 Determine the x coordinate of the point of inflection of f. (3)

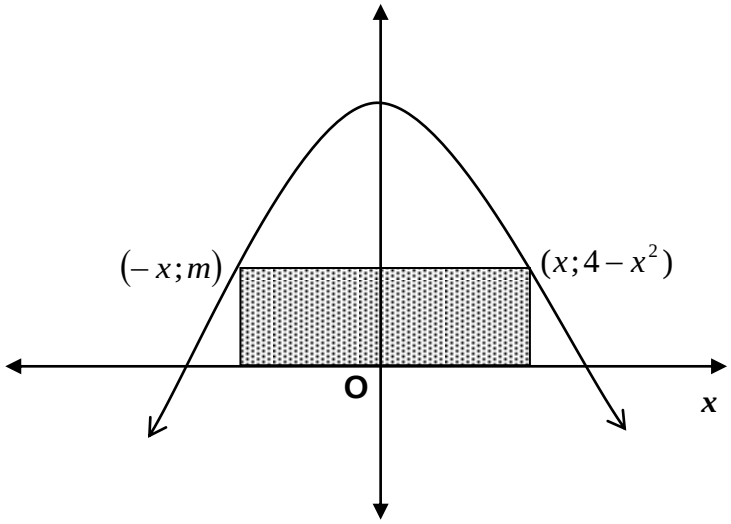
8.4 If $k < 0$, how many real roots will the equation

$x^3 - x^2 - 8x + 12 = k$ have? (2)

[17]

QUESTION 9

9 A rectangle has two vertices on the x – axis and two on the curve with the equation $y = 4 - x^2$ as shown in the sketch below.



9.1 Express m in terms of x. (1)

9.2 Show that the area of the rectangle is given by:

$A = 8x - 2x^3$ (2)

9.3 Determine the area of the largest rectangle which can be drawn in this manner. (5)

[8]

GRAND TOTAL=150

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1} \quad ; \quad r \neq 1 \qquad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \quad ; \quad -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$