Education and Sport Development

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Departement van Onderwys en Sport Ontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

GRADE 12

MATHEMATICS PAPER 1/*WISKUNDE VRAESTEL 1*

MID YEAR EXAMINATION 2019/*HALFJAAREKSAMEN*

MARKING MEMORANDUM /*NASIENMEMORANDUM*

MARKS/*PUNTE*: 150

This marking memorandum consists of 13 pages/*Hierdie memorandum bestaanuit 13 bladsye*

QUESTION/Vraag 1 [26]

1.1.1	$x^2 - x = 0$ $x^2 - x = 0$ $x = 0$ or $x = 1$	✓factors/*faktore* ✓both values of x *Beide x-waardes*(2)
1.1.2	$4 - 10x - 3x^2 = 0$ $3x^2 + 10x - 4 = 0$ $x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-4)}}{2(3)}$ $x = -3{,}69$ or $x = 0{,}36$	✓standard form*standaardvorm* ✓substitution / *subt* ✓✓each value of x *Elke x-waarde* (4)
1.1.3	$\sqrt{11-x} = 1 + x$ $\left(\sqrt{11-x}\right)^2 = (1+x)^2$ $11 - x = x^2 + 2x + 1$ $x^2 + 3x - 10 = 0$ $(x+5)(x-2) = 0$ $x = -5$ or $x = 2$ $\sqrt{11+5} = 1 - 5$ *or* $\sqrt{11-2} = 1 + 2$ $\sqrt{16} \neq -4$ $\quad$ $3 = 3$ $\therefore x = 2$ only	✓$11 - x = x^2 + 2x + 1$ ✓standard form*standaardvorm* ✓factors / *faktore* ✓both values of x / *beide x-waardes* ✓ answer/rejecting $x = -5$ *Antw/verwerp x=-5* (5)
1.1.4	$-(x-4)(x+3) < 0$ $(x-4)(x+3) > 0$ $x < -3$ or $x > 4$	✓$(x-4)(x+3) > 0$ ✓✓correct answer *Korrekteantw* (3)

1.2	$x^2+xy+y^2=7$.....1 $4^{x+2}.8^{y+1}=2^{1-x}$.....2 $2^{2(x+2)}.2^{3(y+1)}=2^{1-x}$ $2^{2x+4+3y+3}=2^{1-x}$ $2x+4+3y+3=1-x$ $3x=-6-3y$.....3 $x=-2-y$ $x^2+xy+y^2=7$ $(-2-y)^2+(-2-y)y+y^2=7$ $4+4y+y^2-2y-y^2+y^2-7=0$ $y^2+2y-3=0$ $(y+3)(y-1)=0$ $y=1$ or $y=-3$ $x=-3$ or $x=1$	✓ $2^{2(x+2)}.2^{3(y+1)}=2^{1-x}$ ✓ $x=-2-y$ ✓substitution / *subst* ✓standard form*standaardvorm* ✓factors/ *faktore* ✓y values/-*waardes* ✓x values/-*waardes* (7)
1.3	$x^2-x=p^2$ $x^2-x-p^2=0$ $\Delta=b^2-4ac$ $\Delta=(-1)^2-4(1)(-p^2)$ $\Delta=1+4p^2$ for equal roots $\Delta=0$ $1+4p^2=0$ $4p^2=-1$ this is impossible since $p^2\geq 0$ *Onmoontlikaangesien*$p^2 \geq 0$	✓Standard form*standaardvorm* ✓substitution /*subst* ✓$\Delta=1+4p^2$ *Virgelykewortels* ✓$1+4p^2=0$ ✓conclusion (5)

QUESTION/Vraag 2 [09]

2.1	$4n+6$ first differences 10 14 18 4 4 $2a=4$ $a=2$	✓first 2 terms of first differences*eerste 2 terme v 1e verskille* ✓second difference / *2e verskil* ✓answer / *antw* (3)

2.2	$3a+b=10$ $3(2)+b=10$ $b=4$ $a+b+c=1$ $2+4+c=1$ $c=-5$ $T_n=2n^2+4n-5$	✓value of *b/ waarde van b* ✓value of *c/ waarde van c* ✓answer /*antw* (3)
2.3	$T_n+T_{n+1}=(2n^2+4n-5)+(2(n+1)^2+4(n+1)-5)$ $=4n^2+12n-4$ $=2(2n^2+6n-2)$ divisible by 2	✓$T_{n+}\ T_{n+1}=.....$ ✓Simplification/ ✓conclusion /*gevolgtrekking* (3)

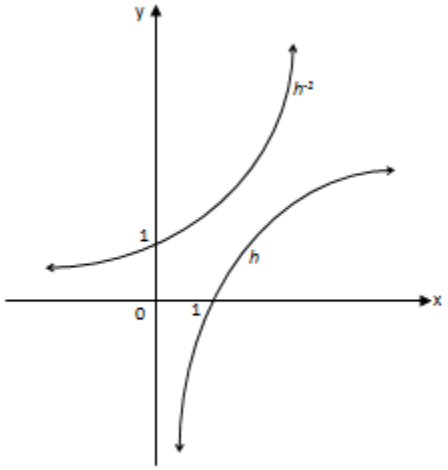
QUESTION/Vraag 3 [23]

3.1.1	$T_n=a+(n-1)d$ $T_n=700+(50-1)\times 450$ $T_n=22750$ $R22\ 750$	✓correct substitution *Korrektesubstitusie* ✓answer / *antw* (2)
3.1.2	First 50m cost R22 750 Additional metres after 50m: $T_n=23450+(n-1)\times 700$ $85050=700n+22750$ $700n=62300$ $n=89$ $\therefore$ Total depth $=50\text{m}+89\text{m}=139\text{m}$	✓$a=23450$ ✓correct substitution*korrektesubstitusie* ✓simplification/*vereenv* ✓answer/*antw* (4)
3.2	$S_n=2n-n^2$ $T_7=S_7-S_6$ $=\left[2(7)-7^2\right]-\left[2(6)-6^2\right]$ $=-11$	✓$T_7=S_7-S_6$ ✓substitution / *subst* ✓answer/ *antw* (3)

3.3	$\sum_{m=0}^{9}\frac{(-1)^{m+1}}{2^m} = -1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\ldots$ $S_{10} = \frac{a\left(1-r^n\right)}{1-r}$ $= \frac{-1\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1-\left(-\frac{1}{2}\right)}$ $= -0{,}67$	✓a ✓r ✓n ✓substitution /*subst* ✓answer /*antw* (5)
3.4.1	$\frac{1+x}{1-x} = \frac{2x+7}{1+x}$ $(1+x)(1+x) = (1-x)(2x+7)$ $1+2x+x^2 = 7-5x-2x^2$ $3x^2+7x-6=0$ $(3x-2)(x+3)=0$ $x=-3$ or $x=\frac{2}{3}$ if $x=-3$ if $x=\frac{2}{3}$ $r=-\frac{1}{2}$ $r=5$ for convergent series: $-1<r<1$ $\therefore x=-3$ only	✓$\frac{1+x}{1-x}=\frac{2x+7}{1+x}$ ✓Standard form *standaardvorm* ✓factors/ *faktore* ✓both values of x *beide x-waardes* ✓both values of r *Beidewaardes van r* ✓$-1<r<1$ ✓$x=-3$ (7)
3.4.2	$S_\infty = \frac{a}{1-r}$ $= \frac{1-x}{1-\frac{1}{2}}$ $= \frac{1-(-3)}{1-\left(-\frac{1}{2}\right)}$ $= \frac{8}{3}$	✓substitution/*subst* ✓answer/ *antw* (2)

QUESTION/Vraag4 [23]

4.1.1	$y = a(x+p)^2 + q$ $y = a(x-2)^2 - 2$ $6 = a(0-2)^2 - 2$ $8 = 4a$ $a = 2$ $f(x) = 2(x-2)^2 - 2$ $f(x) = 2x^2 - 8x + 6$	✓substitution of turning point *subst van draaipunt* ✓subst. (6 ; 0) ✓a = 2 ✓answer / *antw* (4)
4.1.2	$y = 2(x-2)^2 - 2$ $y = 2x^2 - 8x + 8 - 2$ $0 = 2x^2 - 8x + 6$ $0 = x^2 - 4x + 3$ $0 = (x-3)(x-1)$ $x = 1$ or $x = 3$ OR $y = 2(x-2)^2 - 2$ $0 = 2(x-2)^2 - 2$ $(x-2)^2 = 1$ $x - 2 = \pm 1$ $x = 1$ or $x = 3$	✓ $y = 0$ ✓standard form / *standaardvorm* ✓factors /*faktore* ✓both values of x *Beide x-waardes* OR / *OF* ✓ $y = 0$ ✓ $(x-2)^2 = 1$ ✓ $x - 2 = \pm 1$ ✓ both values of x *Beide x-waaardes* (4)
4.1.3	Translation/shift 2 units upward and 2 units to the LHS *Transleer/skuif 2 eenhede op en 2 eenhedena links*	✓ Translation/shift 2 units upward /*2 nabo* ✓2 units to the LHS/*2 na links* (2)
4.1.4	$x \leq 0$ or $x \geq 0$	✓ $x \leq 0$ ✓ $x \geq 0$ (2)
4.2.1	$h(x) = \log_2 x$ $y = 2^x$	✓✓answer / *antw* (2)

4.2.2		h ✓x intercept/*afsnit* ✓shape /*vorm* h^{-1} ✓y intercept / *afsnit* ✓shape / *vorm* (4)
4.2.3	$h(x) = \log_2 x$ $a = \log_2 2$ $a = 1$	✓substitution/*subst* ✓answer / *antw* (2)
4.2.4	$0 < x \leq 2$	✓end points / *interval* ✓correct notation / *notasie* (2)

QUESTION/Vraag5 [18]

5.1	$r(x) = \frac{k}{x+p} + q$ $y = \frac{k}{x-2} - 1$ $-4 = \frac{k}{0-2} - 1$ $-3 = \frac{k}{-2}$ $k = 6$ $r(x) = \frac{6}{x-2} - 1$ $s(x) = a^x + b$ $y = a^x - 1$ $3 = a^2 - 1$ $a^2 = 4$ $a = 2$ $s(x) = 2^x - 1$	✓subst. $x = 2$ and $y = -1$ ✓subst. O (0 ; – 4) ✓equation of *r /vgl. van r* ✓subst ✓equation of *s/ vgl van s* (5)

5.2	$r(x)=\frac{6}{x-2}-1$ $0=\frac{6}{x-2}-1$ $1=\frac{6}{x-2}$ $x-2=6$ $x=8$ B(0 ; 8)	✓$0=\frac{6}{x-2}-1$ ✓$1=\frac{6}{x-2}$ ✓$x=8$ (3)
5.3	$y=-x+c$ *sub* $(2\ ;-1)$ $-1=-2+c$ $c=1$ $y=-x+1$	✓subst. ✓answer / *antw* (2)
5.4	$y=-x+1$ $x=2:\quad y=-2+1=1$ $y=3:\quad x=-3+1=-2$ W(−2 ; 1)	✓value of *x/ waarde van x* ✓value of *y/ waarde van y* (2)
5.5	$0<x<2$ or $x>8$	✓End points / *interval* ✓correct notation/ *notasie* ✓$x>8$ (3)
5.6	$k(x)=s(x)-r(x)$ $k(x)=2^x-1-\left(\frac{6}{x-2}-1\right)$ $k(x)=2^x-1-\frac{6}{x-2}+1$ $k(x)=2^x-\frac{6}{x-2}$ $y\in\Re$	✓removing the brackets *Verwyderhakies* ✓$k(x)=2^x-\frac{6}{x-2}$ ✓answer / *antw* (3)

QUESTION/ Vraag6 [07]

6.1	$A=P(1-in)$ $x=2x(1-0.1n)$ $1=2-0,2n$ $-1=-0,2n$ $n=5$	✓$A=2P$ ✓correct substitution*korrektesubstitusie* ✓removing the brackets/ *verwyderhakies* ✓answer /*antw* (4)

6.2	$A = P(1+i)^n$ $75000 = 42000(1+i)^{5\times12}$ $\frac{75000}{42000} = (1+i)^{60}$ $(1+i)^{60} = \frac{25}{14}$ $1+i = \sqrt[60]{\frac{25}{14}}$ $i = 0{,}009710485$ $\frac{r}{1200} = 0{,}009710485$ $r = 11{,}65\%$	✓ $n = 60$ ✓correct substitution*korrektesubstitusie* ✓ $\sqrt[60]{\frac{25}{14}}$ ✓answer/*antw* (4)
6.3	$(1+i_{eff}) = \left(1+\frac{i_{nom}}{m}\right)^m$ $(1+i_{eff}) = \left(1+\frac{0{,}07}{4}\right)^4$ $i_{eff} = 1{,}071859031 - 1$ $i_{eff} = 7{,}19\%$	✓correct formula*korrekteformule* ✓correct substitution*korrektesubstitusie* ✓answer/ *antw* (3)

QUESTION/ Vraag 7 [18]

7.1.1	$f(x)=-\frac{1}{2}x^2$ $f(x+h)=-\frac{1}{2}(x+h)^2$ $f(x+h)=-\frac{1}{2}(x^2+2xh+h^2)$ $f(x+h)=-\frac{1}{2}x^2-xh-\frac{1}{2}h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-\frac{1}{2}x^2-xh-\frac{1}{2}h^2-\left(-\frac{1}{2}x^2\right)}{h}$ $=\lim_{h\to 0}\frac{-\frac{1}{2}x^2-xh-\frac{1}{2}h^2+\frac{1}{2}x^2}{h}$ $=\lim_{h\to 0}\frac{-xh-\frac{1}{2}h^2}{h}$ $=\lim_{h\to 0}\frac{\left(-x-\frac{1}{2}h\right)h}{h}$ $=\lim_{h\to 0}\left(-x-\frac{1}{2}h\right)$ $=-x$	Penalize 1 mark for incorrect notation/ *penaliseer 1 punt virfoutiewenotasie* ✓correct formula/ *korrekteformule* ✓ $f(x+h)=\frac{1}{2}x^2+xh+\frac{1}{2}h^2$ ✓ $\frac{xh+\frac{1}{2}h^2}{h}$ ✓common factor *h* *Gemene factor h* ✓answer / *antw* (5)
7.1.2	$f(x)=\frac{1}{2}x^2$ $f(-5)=\frac{25}{2}\quad f(3)=\frac{9}{2}$ $m_{av}=\frac{f(-5)-f(3)}{-5-3}$ $=\frac{\frac{25}{2}-\left(\frac{9}{2}\right)}{-8}$ $=-1$	✓ $f(-5)=-\frac{25}{2}$ ✓ $f(3)=-\frac{9}{2}$ ✓method/substitution *Metode/substitusie* ✓answer/ *antw* (4)
7.2	$D_x\left[2x^3-4x-\pi\right]$ $=6x^2-4$	✓$6x^2$ ✓-4 (2)

7.3	$y=\sqrt{x}\left(x-\frac{1}{x}\right)$ $y=x^{\frac{1}{2}}(x-x^{-1})$ $y=x^{\frac{3}{2}}-x^{-\frac{1}{2}}$ $\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$	✓ x^{-1} ✓ $x^{\frac{3}{2}}-x^{-\frac{1}{2}}$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ ✓ $\frac{1}{2}x^{-\frac{3}{2}}$ (4)
7.4	$y=-x^2-4x+12$ $y'=-2x\ -4$ $m=-2x\ -4$ $-14=-2x\ -4$ $-10=-2x$ $x=5$	✓ $y'=-2x\ -4$ ✓ $-14=-2x\ -4$ ✓answer / *antw* (3)

QUESTION/Vraag 8 [16]

8.1	$f(x) = ax^3 + bx + d$ $0 = a(4)^3 + b(4) + d$ substitute (4 ; 0) $0 = 64a + 4b + 16$.....equation 1	✓equation 1/ *vergelyking 1*
	$0 = a(-2)^3 - 2b + d$ substitude (–2 ; 0) $0 = -8a - 2b + 16$.....equation 2 multiply equation 2 by 2 $0 = -16a - 4b + 32$	✓equation 2 *vergelyking 2*
	equation 1 + equation 2 $48a = -48$ $a = -1$	✓method/ *metode*
	$0 = 64(-1) + 4b + 16$ $b = 12$ $f(x) = -x^3 + 12x + 16$	✓substitution/ *subst* (4)
	OR	
	$f'(x) = 3ax^2 + b$ $0 = 3a(-2)^2 + b$ $0 = 12a + b$ $b = -12a$.....equation 1	✓equation 1 *Vergelyking 1*
	$0 = 64a + 4b + 16$.....equation 2 $0 = 64a + 4(-12a) + 16$ $0 = 64a - 48a + 16$ $0 = 16a + 16$ $a = -1$	✓equation 2 *Vergelyking 2*
	$b = -12a$ $b = -12(-1)$ $b = 12$ $f(x) = -x^3 + 12x + 16$	✓method/*metode* ✓substitution/ *subst* (4)
	OR/OF	

	since the zeros of f are $4, -2$ and -2 $f(x) = a(x-4)(x+2)(x+2)$ $f(x) = a(x-4)(x^2+4x+4)$ $f(x) = a(x^3-12x-16)$ substitude $A(0;16)$ $16 = a(-16)$ $a = -1$ $f(x) = -x^3+12x+16$	✓substitution/*subst* ✓ $x^3-12x-16$ ✓substitution of A *Subst van A* ✓$16 = a(-16)$ (4)
8.2	$f(x) = -x^3+12x+16$ $f'(x) = -3x^2+12$ $0 = -3x^2+12$ $x^2-4=0$ $x^2 = 4$ OR $(x-2)(x+2)=0$ $x = \pm 2$ $f(2) = -(2)^3+12(2)+16$ $y = 32$ $Q(2\ ;\ 32)$	✓ $f'(x) = -3x^2+12$ ✓$0 = -3x^2+12$ ✓factors/ $x^2 = 4$ /*faktore* ✓both values of x *Beide x-waardes* ✓ value of y *Waaarde van y* (5)
8.3	$f'(x) = -3x^2+12$ $f''(x) = -6x$ $0 = -6x$ $x = 0$ $y = 16$ $(0;16)$ $\therefore (0;-16)$ **OR/OF** $f(x) = -x^3-12x+16$ $t(x) = -f(x) = x^3-12x-16$ $t'(x) = 3x^2-12$ $t''(x) = 6x$ $0 = 6x$ $x = 0$ $t(0) = -16$ $\therefore (0\ ;-16)$	✓ $f''(x) = -6x$ ✓$0 = -6x$ ✓ $x = 0$ ✓answer /*antw* (4) ✓ $t(x) = x^3-12x-16$ ✓ $f''(x) = 6x$ ✓$0 = 6x$ ✓answer/ *antw* (4)
8.4	$x < -2$ *or* $2 \le x < 4$	✓ $x < -2$ ✓✓ $2 \le x < 4$ (3)

QUESTION / Vraag9 [7]

9.1	$P(A \text{ and } B) = P(A) \times P(B)$ $= 0,4 \times 0,5$ $= 0,2$	✓substitution / *subst* ✓answer / *antw* (2)
9.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,4 + 0,5 - 0,2$ $= 0,7$	✓substitution / *subst* ✓answer/ *antw* (2)
9.3	$P(\text{not } A \text{ and not } B) = 1 - P(A \text{ or } B)$ $= 1 - 0,7$ $= 0,3$	✓formula / *formule* ✓substitution / *subst* ✓answer / *antw* (3)