Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sport Ontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD12

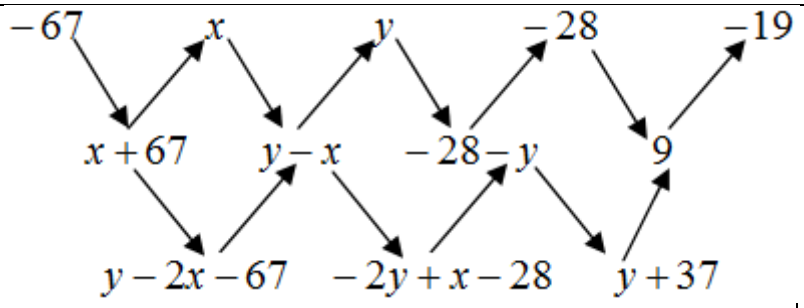
MATHEMATICS PAPER 1/VRAESTEL 1 WISKUNDE

MEMORANDUM

MID YEAR EXAMINATION 2018 HALFJAAREKSAMEN

MARKS / PUNTE: 150

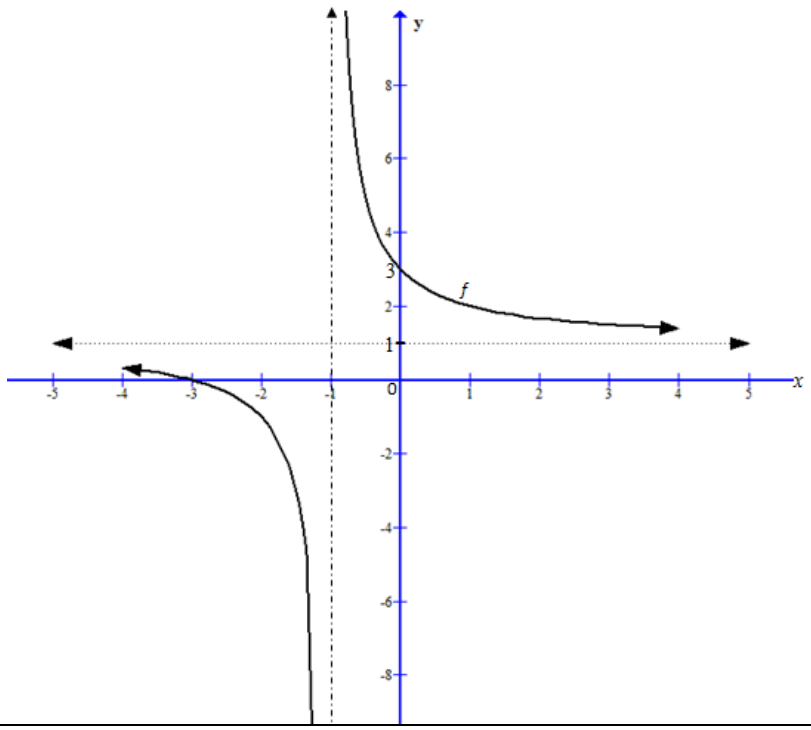
QUESTION/VRAAG 1		
1.1.1	$x^2 - 5x = -6$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2$ or $x = 3$	✓standard form/*standaardvorm* ✓factors/faktore ✓both values of *x*/ *beide x-waardes* (3)
1.1.2	$3x^2 - 4x - 2 = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{4 \pm \sqrt{40}}{6}$ $x = 1{,}72 \ \ or \ \ x = -0{,}39$	✓correct substitution / *korrektesubstitusie* ✓ $\sqrt{40}$ ✓✓each value of *x* *Elke x-waarde* (5)
1.1.3	$(3x+1)(x-4) > 0$ $\therefore x < -\frac{1}{3} \ \ or \ \ x > 4$	✓end points/ *waardes* ✓correct notation *korrektenotasie* (2)
1.2	$\frac{4^x}{2^y} = 256$ $\frac{2^{2x}}{2^y} = 2^8$ $2^{2x-y} = 2^8$ $\therefore 2x - y = 8$ $x^2 - xy + y^2 = 19$ $y = 2x - 8$ $x^2 - x(2x-8) + (2x-8)^2 = 19$ $x^2 - 2x^2 + 8x + 4x^2 - 32x + 64 - 19 = 0$ $3x^2 - 24x + 45 = 0$ $x^2 - 8x + 15 = 0$ $(x-5)(x-3) = 0$ $x = 3$ or $x = 5$ $y = 2x - 8$ $y = 2(3) - 8$ $\quad$ $y = 2(5) - 8$ $y = -2$ $\quad$ or $\quad$ $y = 2$	✓ $2x - y = 8$ ✓making *y* subject/ *maak y onderwerp* ✓correct substitution / *korrektesubstitusie* ✓simplification / *vereenvoudiging* ✓standard form/ *standaardvorm* ✓factors / *faktore* ✓both *x* values / *beide x-waardes* ✓both *y* values / *beide y-waardes* (8)

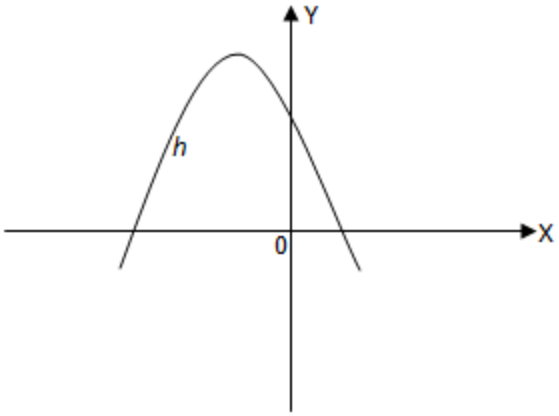
1.3	$x^5 = x^2 \times x^2 \times x$ $= 3 \times 3 \times (\pm\sqrt{3})$ $= \pm 9\sqrt{3}$ *OR* $x^2 = 3$ $x^{2\times\frac{5}{2}} = 3^{\frac{5}{2}}$ $x^5 = \sqrt{243}$ $x^5 = \pm 9\sqrt{3}$	✓exponential law/ *eksponentwet* ✓substitution / *substitusie* ✓answer / *antw* ✓exponential law/ *Eksp wet* ✓ $x^5 = \sqrt{243}$ ✓answer /*antw* (3)
QUESTION/VRAAG 2		
2.1	 $y - 2x - 67 = y + 37$ $-2x = 104$ $x = -52$ $y - 2x - 67 = -2y + x - 28$ $3y - 3x = 39$ $y - x = 13$ $y - (-52) = 13$ $y = -39$	✓first differences / *Eersteverskille* ✓second differences / *Tweedeverskille* ✓equating second differences / *tweedeverskillegelykste l* ✓*x*-value / *x-waarde* ✓ equating second differences/ *tweedeverskillegelykste l* ✓*y*-value /*y-waarde* (6)

2.2	$-67 \quad -52 \quad -39 \quad -28 \quad -19$ $15 \quad 13 \quad 11$ $-2 \quad -2$ $2a = -2$ $a = -1$ $3a + b = 15$ $3(-1) + b = 15$ $b = 18$ $a + b + c = -67$ $-1 + 18 + c = -67$ $c = -84$ $T_n = -n^2 + 18n - 84$	✓value of a *Waarde van a* ✓value of b *Waarde van b* ✓value of c *Waarde van c* ✓answer *antwoord* (4)
2.3	$T_n > 0$ $-n^2 + 18n - 84 > 0$ $n^2 - 18n + 84 < 0$ no Solution $\therefore$ the sequence will never contain a positive term *Geenoplossing* *d.w.s. die getalpatroonsalnooitpositiewe term hênie*	✓$-n^2 + 18n - 84 > 0$ ✓$n^2 - 18n + 84 < 0$ ✓conclusion *gevolgtrekking* (3)

QUESTION/VRAAG 3		
3.1	$T_4 = 24 \qquad T_9 = 768$ $\frac{T_9}{T_4} = r^{9-4}$ $\frac{768}{24} = r^5$ $32 = r^5$ $2^5 = r^5$ $r = 2$ $\frac{T_4}{T_1} = r^3 \qquad or \qquad \frac{T_9}{T_1} = r^8$ $\frac{24}{a} = 2^3 \qquad \frac{768}{a} = 2^8$ $a = \frac{24}{8} \qquad a = \frac{768}{256}$ $a = 3 \qquad a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12; ..... Or $T_4 = 24 \qquad T_9 = 768$ $T_n = ar^{n-1}$ $T_4 = ar^3 = 24$.....equation1 $T_9 = ar^8 = 768$.....equation2 $\frac{\text{equation2}}{\text{equation1}}$ $ar^8 = 768$ $ar^3 = 24$ $r^5 = 32$ $r^5 = 2^5$ $r = 2$ $ar^3 = 24 \qquad or \qquad ar^8 = 768$ $a \cdot 2^3 = 24 \qquad a \cdot 2^8 = 768$ $a = 3 \qquad a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12;	✓$\frac{768}{24} = r^5$ ✓simplification / *vereenvoudiging* ✓r ✓a ✓T_2& T_3 (5) ✓2 equations/ *2 vergelykings* ✓$r^5 = 32$ ✓r ✓a ✓ T_2& T_3 (5)

3.2.1	$T_{12} = S_{12} - S_{11}$ $= 324 - 275$ $= 49$	✓ $T_{12} = S_{12} - S_{11}$ ✓substitution / *subst* ✓answer / *antw* (3)
3.2.2	$T_n = S_n - S_{n-1}$ $= 2n^2 + 3n - \left[2(n-1)^2 + 3(n-1)\right]$ $= 2n^2 + 3n - \left[2n^2 - 4n + 2 + 3n - 3\right]$ $= 2n^2 + 3n - 2n^2 + n + 1$ $= 4n + 1$	✓ $T_n = S_n - S_{n-1}$ ✓substitution / *substitusie* ✓simplification / *vereenvoudiging* ✓answer / *antw* (4)
3.3	$\sum_{n=2}^{18}(2n - 1)$ $3 + 5 + 7 + \ldots$ $a = 3$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $d = 2$ $\quad S_{17} = \frac{17}{2}\left[2 \cdot 3 + (17-1)2\right]$ $n = 17$ $\quad = 323$ $S_n = ?$	✓ $n = 17$ ✓ a&d ✓answer / *antw* (3)
3.4	$S_\infty = \frac{40}{3}$ $S_\infty = \frac{a}{1-r} = \frac{40}{3}$ $a = \frac{40}{3}(1-r)$ $a = \frac{40}{3}\left(1 - \frac{5}{2a}\right)$ $a = \frac{40}{3} - \frac{100}{3a}$ $3a^2 = 40a - 100$ $3a^2 - 40a + 100 = 0$ $(3a - 10)(a - 10) = 0$ $a = \frac{10}{3}$ or $a = 10$ $T_2 = \frac{5}{2}$ $T_n = ar^{n-1}$ $T_2 = ar = \frac{5}{2}$ $r = \frac{5}{2a}$	✓ $\frac{a}{1-r} = \frac{40}{3}$ ✓ $ar = \frac{5}{2}$ ✓ $r = \frac{5}{2a}$ ✓substitution / *substitusie* ✓standard form / *standaardvorm* ✓factors/*faktore* ✓ answer(both values of *a*)/ *Antw (beidewaardes van a)* (7)

QUESTION/VRAAG 4		
4.1	$f(x)=\frac{x+3}{x+1}$ $y=0:\quad 0=\frac{x+3}{x+1}$ $0=x+3$ $x=-3$ $x=0:\quad y=\frac{0+3}{0+1}$ $y=3$	✓y= 0 ✓x = -3 ✓y = 3 (3)
4.2	$f(x)=\frac{x+3}{x+1}$ $f(x)=\frac{x+2+1}{x+1}$ $f(x)=\frac{2}{x+1}+\frac{x+1}{x+1}$ $f(x)=\frac{2}{x+1}+1$	✓ $f(x)=\frac{x+2+1}{x+1}$ ✓ $\frac{2}{x+1}+\frac{x+1}{x+1}$ (3)
4.3.1	$x=-1$	✓answer/*antw* (1)
4.3.2	$y=1$	✓answer /*antw* (1)
4.4		✓asymptotes *asimptote* ✓x-intercept/*afsnit* ✓y-intercept/*afsnit* ✓correct shape *Korrektevorm* (4)
4.5	$-1<x<0$	✓end points/*interval* ✓correct notation *Korrektenotasie* (2)

4.6	$\sum_0^3 f(x)$ $\sum_0^3 \frac{x+3}{x+1} = \frac{0+3}{0+1} + \frac{1+3}{1+1} + \frac{2+3}{2+1} + \frac{3+3}{3+1}$ $= 3 + 2 + \frac{5}{3} + \frac{3}{2}$ $= \frac{49}{6}$ $(\text{or } = 8{,}17)$	✓term 1 & 2 ✓term 3 & 4 ✓answer/*antw* (3)
QUESTION/VRAAG 5		
5.1.1	$y = \log_2 x$	✓✓answer/*antw* (2)
5.1.2	$f(x) = 2^x \quad y = 3 \quad f^{-1}(x) = y = \log_2 x$ $3 = 2^x \qquad 3 = \log_2 x$ $x = \frac{\log 3}{\log 2} \qquad 2^3 = x$ $x_A = 1{,}58 \qquad x_B = 8$ $AB = x_B - x_A$ $AB = 8 - 1{,}58$ $AB = 6{,}42$	✓$3 = 2^x$ ✓$x_A = 1{,}58$ ✓ $3 = \log_2 x$ ✓ $x_B = 8$ ✓answer/*antw* (5)
5.1.3	$m_{av} = \frac{f(a) - f(b)}{a - b}$ $= \frac{256 - 2{,}9897}{8 - 1{,}58}$ $= 39{,}41$	✓numerator /*teller* ✓method/*metode* ✓answer.*antw* (3)
5.1.4	$x > 0, \quad x \in \Re$	✓answer/*antw* (1)
5.2	a < 0 concave down/ *konkaafnaonder* b < 0 shift to left / *skuifna links* c > 0 y-int above 0 / *y-afsnitbokant 0* 	✓two x intercepts/ *Twee x-afsnitte* ✓with different signs *Met versk. Tekens* ✓turning points in quadrant 2 / *draaipt in 2de kwadrant* ✓concave down/ *Konkaafnaonder* ✓y intercept above the *x* axis *y-afsnitbo die x-as* (5)

QUESTION/VRAAG 6		
6.1.1	$1+i^{eff} = (1+\frac{i^n}{n})^n$ $i^{eff} = (1+\frac{0,11}{12})^{15} - 1$ $i^{eff} = 1,146683.... - 1$ $= 0,146683...$ $i^{eff} = 14,67\%$	✓substitution into correct formula *Subt in korrekteformule* ✓$i = \frac{0,18}{12}$ ✓answer / *antw* (3)
6.1.2	$P = 650000$ $n = 15\times12 = 180$ $i = \frac{11}{1200}$ $x = ?$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $x = \frac{Pi}{\left[1-(1+i)^{-n}\right]}$ $x = \frac{650000\times\frac{11}{1200}}{1-\left(1+\frac{11}{1200}\right)^{-180}}$ $x = 7387.88$	✓ substitution into correct formula *Subst in korrekteformule* ✓P = 500 000 ✓ n = 180 ✓answer /*antw* (4)
6.1.3	$n = 8\times12 = 96$ Balance = A – F $= 650000\left(1+\frac{11}{1200}\right)^{96} - \frac{7387,88\left[\left(1+\frac{11}{1200}\right)^{96}-1\right]}{\frac{11}{1200}}$ $= 43147366$ **OR** $n = (15-8)\times12 = 84$ $P = \frac{7387,88\left[1-\left(1+\frac{11}{1200}\right)^{-84}\right]}{\frac{11}{1200}}$ $P = 43147364$	✓substitute into correct formula/ *Subst in korrekteformule* ✓n = 24 ✓$\frac{0,11}{12}$ ✓answer/ *antw* (4) ✓substitute into correct formula/ *Subst in korrekteformule* ✓ n = -84 ✓$\frac{0,11}{12}$ ✓answer / *antw* (4)

6.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\frac{Fi}{x} + 1 = (1+i)^n$ $\frac{50000 \times \frac{3}{200}}{2300} + 1 = \left(1 + \frac{3}{200}\right)^n$ $\frac{61}{46} = 1{,}015^n$ $n = \frac{\log \frac{61}{46}}{\log 1{,}015}$ $n = 18{,}96$ Accept $n = 19$	✓substitute into correct formula/ *Subst in korrekteformule* ✓simplification/ *vereenvoudiging* ✓ $\frac{61}{46} = 1{,}015^n$ ✓application of log law / *toepassing van logwet* ✓answer / *antw* (4)
QUESTION /VRAAG 7		
7.1.1	$f(x) = x^2 - 6x$ $f(x+h) = x^2 + 2xh + h^2 - 6x - 6h$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h - \left(x^2 - 6x\right)}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h - x^2 + 6x}{h}$ $= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$ $= \lim_{h \to 0} \frac{(2x + h - 6)h}{h}$ $= \lim_{h \to 0} (2x + h - 6)$ $= 2x - 6$	✓correct notation/ *Korrektenotasie* ✓correct substitution/ *korrektesubst* ✓simplification/ *vereenvoudiging* ✓common factor *h*/ *Gemene factor h* ✓answer / *antw* (5)

7.1.2	$f(x) = x^2 - 6x$ $f(-2) = (-2)^2 - 6(-2) = 16$ $f(1) = (1)^2 - 6(1) = -5$ $m_{AV} = \frac{f(1) - f(-2)}{1 - (-2)}$ $= \frac{-5 - 16}{3}$ $= -\frac{21}{3}$ $= -7$	✓method / *metode* ✓ $f(2) = 16$ ✓ $f(1) = -5$ ✓answer/ *antw* (4)
7.1.3	$f'(x) = 2x - 6$ $m = 2x - 6$ $-4 = 2x - 6$ $2 = 2x$ $x = 1$ $f(1) = 1^2 - 6(1)$ $y = -5$ $y - y_1 = m(x - x_1)$ $y + 5 = -4(x - 1)$ $y = -4x + 4 - 5$ $y = -4x - 1$ substitute $(a\ ;\ 0)$ $0 = -4a - 1$ $a = -\frac{1}{4}$	✓ $-4 = 2x - 6$ ✓ $x = 1$ ✓ $y = -5$ ✓substitution into straight line formula *Subst in reguitlynformule* ✓equation of tangent / *vgl van raaklyn* ✓answer/ *antw* (6)

7.1.4	$f'(x) = 2x - 6$ $f''(x) =$	
7.2	$D_x\left[\sqrt[3]{x} + x^2 + 4x\right]$ $= D_x\left[x^{\frac{1}{3}} + x^2 + 4x\right]$ $= \frac{1}{3}x^{-\frac{2}{3}} + 2x + 4$	✓ $x^{\frac{1}{3}}$ ✓ $\frac{1}{3}x^{-\frac{2}{3}}$ ✓ $2x+4$ (3)
7.3.1	$xy = 5$ $y = \frac{5}{x}$ $y = 5x^{-1}$ $\frac{dy}{dx} = -5x^{-2}$	✓ $y = 5x^{-1}$ ✓answer / *antw* (2)
7.3.2	$y = \frac{2x^3 - x}{\sqrt{x}}$ $y = \frac{2x^3 - x}{x^{\frac{1}{2}}}$ $y = \frac{2x^3}{x^{\frac{1}{2}}} - \frac{x}{x^{\frac{1}{2}}}$ $y = 2x^{\frac{5}{2}} - x^{\frac{1}{2}}$ $\frac{dy}{dx} = 5x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$	✓ $x^{\frac{1}{2}}$ ✓ $y = 2x^{\frac{5}{2}} - x^{\frac{1}{2}}$ ✓ $5x^{\frac{3}{2}}$ ✓ $-\frac{1}{2}x^{-\frac{1}{2}}$ (4)
QUESTION/VRAAG 8		
8.1.1	$(x-2)(ax^2+bx+c)$ $(x-2)(x^2+bx-6)$ $-2bx - 6x = -8x$ $-2b - 6 = -8$ $-2b = -2$ $b = 1$ $0 = (x-2)(x^2+x-6)$ $0 = (x-2)(x-2)(x+3)$ $x = 2 \quad or \quad x = -3$ A(–3 ; 0)	✓method/*metode* ✓ $x^2 + x - 6$ ✓factors/*faktore* ✓values of *x/ waardes van x* ✓coordinates of A/ *Koördiante van A* (5)

8.1.2	$f(x) = x^3 - x^2 - 8x + 12$ $f'(x) = 3x^2 - 2x - 8$ $0 = 3x^2 - 2x - 8$ $0 = (3x + 4)(x - 2)$ $x = 2 \quad or \quad x = -\frac{4}{3}$ $x_B = -\frac{4}{3}$	✓ $f'(x) = 3x^2 - 2x - 8$ ✓correct factors/ *korrektefaktore* ✓both values of x / *beidewaardes van x* ✓answer/ *antw* (4)
8.1.3	$x < -\frac{4}{3}$ or $x > 2$	✓end points/ *waardes* ✓notation/ *notasie* ✓or / *of* (3)
8.1.4	$f'(x) = 3x^2 - 2x - 8$ $f''(x) = 6x - 2$ $0 = 6x - 2$ $x = \frac{1}{3}$	✓second derivative/ *tweedeafgeleide* ✓ $f''(x) = 0$ ✓answer/*antw* (3)
8.1.5	ONE real root / *EEN reëlewortel*	✓answer/ *antw* (2) [17]
QUESTION/VRAAG 9		
9.1	$m = 4 - x^2$	✓answer/*antw* (1)
9.2	$A = l \times b$ $A = 2x(4 - x^2)$ $A = 8x - 2x^3$	✓$2x$ ✓$4 - x^2$ (2)
9.3	$A = 8x - 2x^3$ $A' = 8 - 6x^2$ $0 = 8 - 6x^2$ $6x^2 = 8$ $x = \frac{2}{\sqrt{3}}$ $A = 8\left(\frac{2}{\sqrt{3}}\right) - 2\left(\frac{2}{\sqrt{3}}\right)^3$ $A = \frac{32\sqrt{3}}{9}$ *Accept* $A = 6{,}16$	✓ $A' = 8 - 6x^2$ ✓ $0 = 8 - 6x^2$ ✓ $x = \frac{2}{\sqrt{3}}$ ✓substitution/ *subst* ✓answer/*antw* (5) **[8]**