Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sport Ontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

GRADE 12

MATHEMATICS P2 / WISKUNDE V2

MID-YEAR EXAMINATION 2018 HALFJAAREKSAMEN

MARKING GUIDELINE/ MEMORANDUM

TOTAL MARKS/TOTALE PUNTE : 150

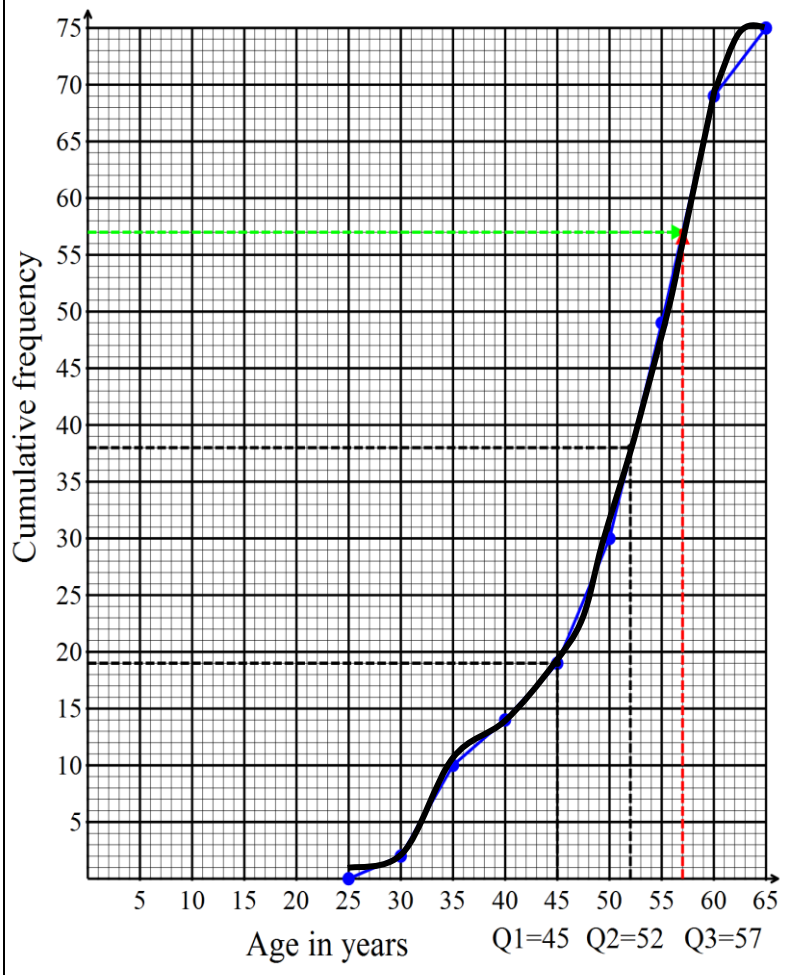
This marking guideline consist of 14 pages. / Hierdie memorandum bestaan uit 14 bladsye.

QUESTION/VRAAG 1

1.1

Age / *Ouderdom*	Frequency / *Frekwensie*	Cumulative frequency /*Kum. Frekwensie*
25 < A ≤30	2	2
30 < A ≤35	8	10
35 < A ≤40	**4**	**14**
40 < A ≤45	**5**	**19**
45 < A ≤50	**11**	**30**
50 < A ≤55	**19**	**49**
55 < A ≤60	**20**	**69**
60 < A ≤65	6	**75**

1.1.1



✓ correct grounding
Korrek gegrond

✓ correct shape (smooth curve of points joined)
Punte verbind in gladde vorm

✓✓ all points correct
Alle punte korrek

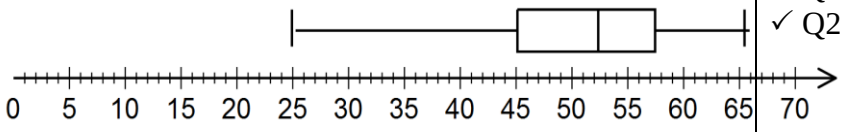
✓ min 6 points correct
Minstens 6 punte reg

(4)

1.1.2

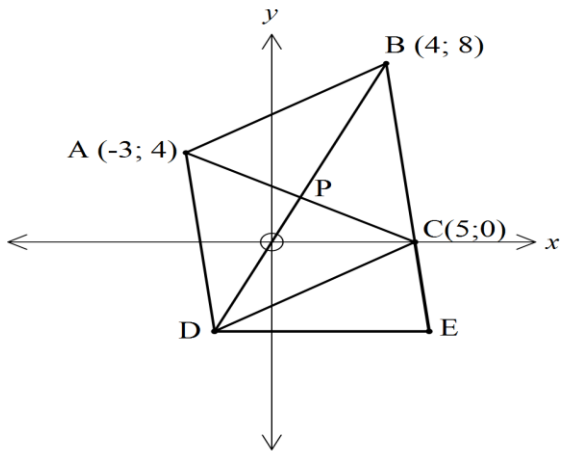
Median / *mediaan* = 52 **(** Accept/*aanvaar* 51≤ Med ≤ 53)
for dotted lines from 38 across to ogive and down to age
vir stippellyne vanaf 38 dwars na ogief en af na ouderdom

✓
✓ Answer *antw*
(2)

1.1.3	% of teachers older than $50\text{yrs} = \frac{75-30}{75} \times 100$ % *onderwysers ouer as 50 jr* $= \frac{45}{75} \times 100$ $= 60\%$	✓ 75 ✓ 30 ✓ method / *metode* ✓ Answer / *antw* (4)
1.1.4	✓ max and min ✓ Q1and Q3 and ✓ Q2 and shape of box  	✓ min and max *Min / maks* ✓ Q1 and /*en* Q3 ✓ Q2 & *houer* (3)
1.1.5	Skewed to the left or negatively skewed *Skeef na links of negatief skeef*	✓ Answer / *antw* (1)
1.2	**68 12 44 71 27 86 52**	
1.2.1	Mean/ *gemiddeld* = 51, 43	✓ ✓ Answer / *antw* (2)
1.2.2	Std deviation/ *standaardafwyking* = 24, 08	✓ ✓ Answer / *antw* (2)
1.2.3	(27,35 – 75,51) 68; 44; 71; 86; 52 5 learners are within one std deviation *5 leerders binne een std afwyking*	✓ ✓ Answer/ *antw* (2)

[20]

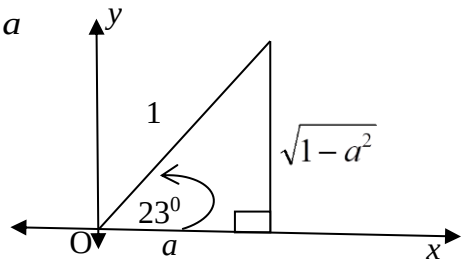
QUESTION/VRAAG 2

<table>
<tr><td></td><td colspan="2"> 
</td></tr>
<tr><td>2.1</td><td>$m_{BCE} = \frac{8-0}{4-5}$
$m_{BCE} = -8$
$y - y_1 = m(x - x_1)$
$y - 0 = -8(x - 5)$
$\therefore y = -8x + 40$</td><td>✓ $m_{BCE} = \frac{8-0}{4-5}$
✓ $m_{BCE} = -8$
✓ $y - 0 = -8(x - 5)$
✓ $y = -8x + 40$
(4)</td></tr>
<tr><td>2.2</td><td>P is the midpoint of AC because diagonals of a parm bisect each other. / P is midpt v AC, hoeklyne halveer
$P\left(\frac{-3+5}{2}; \frac{4+0}{2}\right)$
$P\left(\frac{2}{2}; \frac{4}{2}\right)$
P(1; 2)</td><td>✓ Method/ metode
✓ Answer/ antw
(2)</td></tr>
<tr><td>2.3</td><td>D(– 2; – 4)</td><td>✓ – 2
✓ – 4
(2)</td></tr>
<tr><td>2.4</td><td>$m_{AC} \times m_{BD} = -\frac{1}{2} \times 2 = -1$
Diagonals of a parm bisect at 90°
Hoeklyne van $\parallel^m$ sny $\perp$
$\therefore$ ABCD is a rhombus / ABCD is 'n ruit
NB: must mention that ABCD is a parm
NB moet noem dat ABCD $\parallel^m$ is</td><td>✓ $m_{AC} \times m_{BD} = -\frac{1}{2} \times 2 = -1$
✓ Diagonals of parm
Hoeklyne van $\parallel^m$
✓ bisect at 90°
sny by 90°
(3)</td></tr>
</table>

2.5	$\hat{ACB} = \alpha - \beta$ $\tan\alpha = m_{AC} = -\frac{1}{2}$ $\therefore \alpha = 180^\circ - 26{,}57^\circ$ $\alpha = 153{,}43^\circ$ $\tan\beta = m_{BC} = -8$ $\therefore \beta = 180^\circ - 82{,}87^\circ$ $\beta = 56{,}31^\circ$ $A\hat{C}B = \alpha - \beta$ $A\hat{C}B = 153{,}43^\circ - 97{,}13^\circ$ $A\hat{C}B = 56{,}31^\circ$ OR $AB = BC = \sqrt{65}$ $AC = \sqrt{80}$ $\left(\sqrt{65}\right)^2 = \left(\sqrt{65}\right)^2 + \left(\sqrt{80}\right)^2 - 2\left(\sqrt{65}\right)\left(\sqrt{80}\right)\cos A\hat{C}B$ $\cos A\hat{C}B = 0{,}5547$ $A\hat{C}B = 56{,}31^\circ$	✓ $\tan\alpha = m_{AC} = -\frac{1}{2}$ ✓ $\alpha = 153{,}43^\circ$ ✓ $\tan\beta = m_{BC} = -8$ ✓ $\beta = 56{,}31^\circ$ ✓ $A\hat{C}B = 56{,}31^\circ$ ✓ $AB = BC = \sqrt{65}$ ✓ $AC = \sqrt{80}$ ✓ Subst on formula *Subst in formule* ✓ $\cos A\hat{C}B = 0{,}5547$ ✓ $A\hat{C}B = 56{,}31^\circ$ (5)
2.6	At E: $y = -4$ $y = -8x + 40$ $-4 = -8x + 40$ $x = 5\frac{1}{2}$ D$(-2; -4)$ $\therefore$ DE = $5\frac{1}{2} - (-2) = 7\frac{1}{2}$ units / *eenhede* CA applies to E from equation in 1.1 and to D in 1.2 *VA toegepas op E uit vgl in 1.1 en D uit 1.2*	✓ $-4 = -8x + 40$ ✓ $x = 5\frac{1}{2}$ ✓ DE = $7\frac{1}{2}$ units / *eenhede* (3)
2.7	Area of ΔABC = $\frac{1}{2}AC.BC.\sin A\hat{C}B$ = $\frac{1}{2}\sqrt{80}\times\sqrt{65}\sin 56{,}31^\circ$ = 30 units^2 /*eenhede²* OR/*OF* AC = $\sqrt{45}$ Area of ΔABC = $\frac{1}{2}AC \times BP$ = $\frac{1}{2}\sqrt{80}\times\sqrt{45}$ = 30 units^2 / *eenhede²*	✓ $\frac{1}{2}AC.BC.\sin A\hat{C}B$ ✓ $\frac{1}{2}\sqrt{80}\times\sqrt{65}\sin 56{,}31^\circ$ ✓ 30 units^2 /*eenhede²* ✓ AC = $\sqrt{45}$ ✓ $\frac{1}{2}\sqrt{80}\times\sqrt{45}$ ✓ 30 units^2 /*eenhede²* (3)

[22]

QUESTION/VRAAG 3

3.1	$\frac{\cos(90^\circ + B).\sin(450^\circ + B)}{\cos(180^\circ + B).\cos(B-180^\circ)}$ $= \frac{-\sin B.\sin(90^\circ + B)}{-\cos B.(-\cos B)}$ $= \frac{-\sin B.\cos B}{-\cos B.(-\cos B)}$ $= \frac{-\sin B.}{\cos B.}$ $= -\tan B$	✓ – sin B ✓ – cos B ✓ – cos B ✓ cos B ✓ – tan B (5)
3.2	$\frac{3\tan 123^\circ.\cos 417^\circ}{\cos 147^\circ.\sin 270^\circ}$ $= \frac{3\tan(180^\circ - 57^\circ).\cos 57^\circ}{\cos(90^\circ + 57^\circ).(-1)}$ $= \frac{-3\tan 57^\circ.\cos 57^\circ}{\sin 57^\circ.(-1)}$ $= -3\tan 57^\circ \times \frac{1}{\tan 57^\circ}$ $= -3$	✓ $-3\tan 57^\circ$ ✓ $\cos 57^\circ$ ✓ $\sin 57^\circ$ ✓ -1 ✓ $\frac{1}{\tan 57^\circ}$ ✓ answer/*antw* (6)
3.3	$\cos 23^\circ = a$  	✓ sketch including values *Skets insluitend waardes* ✓ $\sqrt{1-a^2}$
3.3.1	$\tan 23^\circ = \frac{\sqrt{1-a^2}}{a}$	✓ answer / *antw* (3)
3.3.2	$\sin 46^\circ = \sin 2(23^\circ)$ $= 2\sin 23^\circ \cos 23^\circ$ $= 2 \times \sqrt{1-a^2} \times a$ $= 2a\sqrt{1-a^2}$	✓ expansion/*uitbreiding* ✓ substitution/*substitusie* ✓ answer/*antw* (3)
3.3.3	$\cos 44^\circ = \cos(67^\circ - 23^\circ)$ $= \cos 67^\circ.\cos 23^\circ + \sin 67^\circ.\cos 23^\circ$ $= \sqrt{1-a^2} \times a + a \times \sqrt{1-a^2}$ $= 2a\sqrt{1-a^2}$	✓ expansion/*uitbreiding* ✓ substitution/*substitusie* ✓ answer/*antw* (3)

[20]

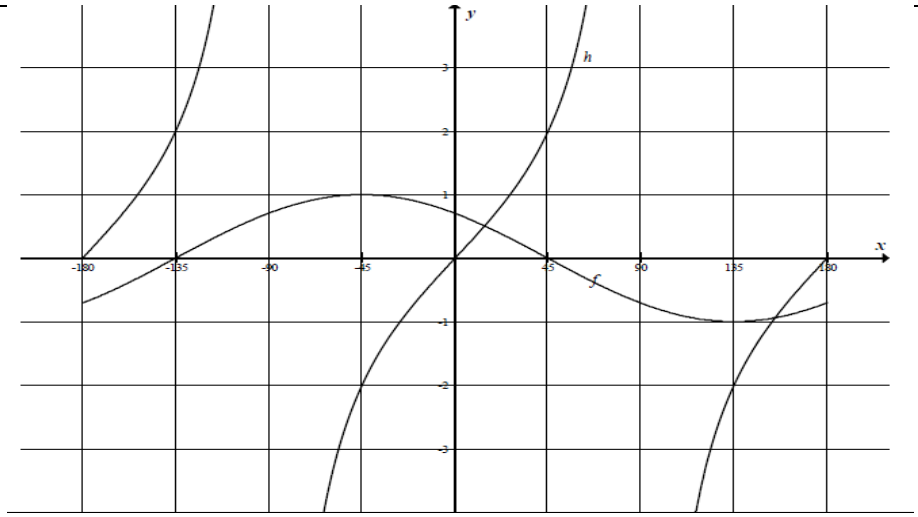
QUESTION/VRAAG 4

4.1.1	$\sin 105^\circ = \sin(60^0 + 45^0)$ $= \sin 60^0 \cos 45^0 + \cos 60^0 \sin 45^0$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	✓ $\sin(60^0 + 45^0)$ ✓ $\sin 60^0 \cos 45^0 + \cos 60^0 \sin 45^0$ ✓ substitution/*substitusie* ✓ answer/*antw* (4)
4.1.2	$\cos 69^\circ.\cos 9^\circ + \cos 81^\circ.\cos 21^\circ$ $= \cos 69^\circ.\cos 9^\circ + \sin 9^\circ.\sin 69^\circ$ $= \cos(69^0 - 9^0)$ $= \cos 60^0$ $= \frac{1}{2}$ OR/OF $\cos 69^\circ.\cos 9^\circ + \cos 81^\circ.\cos 21^\circ$ $= \sin 21^\circ.\cos 9^\circ + \sin 9^\circ.\cos 21^\circ$ $= \sin(21^0 + 9^0)$ $= \sin(30^0)$ $= \frac{1}{2}$	✓ changing to co-functions *Verander na ko-funksies* ✓ $\cos(69^0 - 9^0)$ ✓ $\cos 60^0$ ✓ answer/ *antw* ✓ changing to co-functions *Verander na ko-funksies* ✓ $\sin(21^\circ + 9^\circ)$ ✓ $\sin 30^0$ ✓ answer/*antw* (4)
4.2	$\frac{\sin 2x - \cos x}{1 - \cos 2x - \sin x} = \frac{\cos x}{\sin x}$ $LHS = \frac{\sin 2x - \cos x}{1 - \cos 2x - \sin x}$ $= \frac{2\sin x \cos x - \cos x}{1 - (1 - 2\sin^2 x) - \sin x}$ $= \frac{\cos x(2\sin x - 1)}{1 - 1 + 2\sin^2 x - \sin x}$ $= \frac{\cos x(2\sin x - 1)}{2\sin^2 x - \sin x}$ $= \frac{\cos x(2\sin x - 1)}{\sin x(2\sin x - 1)}$ $= \frac{\cos x}{\sin x}$ $\therefore LHS = RHS$ $LK = RK$	✓ $2\sin x \cos x$ ✓ $1 - 2\sin^2 x$ ✓ $\cos x(2\sin x - 1)$ ✓ $1 - 1 + 2\sin^2 x - \sin x$ ✓ $\sin x(2\sin x - 1)$ (5)
4.3	$2\cos 2x + 1 = 0$; where $x \in [-180^\circ; 0^\circ]$	

	$2\cos 2x+1=0$ $\cos 2x=-\frac{1}{2}$ $2x=\pm 120^0+k.360^0$ $x=\pm 60^0+k.180^0; k\in Z$ SS $\{-120^0;\ -60^0\}$ **OR / OF** $2\cos 2x+1=0$ $\cos 2x=-\frac{1}{2}$ Acute angle = 60^0 2^{nd} quad: $2x=180^0-60^0+k.360^0$ $2x=120^0+k.360^0$ $x=60^0+k.180^0; k\in Z$ 3^{rd} quad: $2x=180^0+60^0+k.360^0$ OR $2x=240^0+k.360^0$ $x=120^0+k.180^0; k\in Z$ SS $\{-120^0;\ -60^0\}$	✓ $\cos 2x=-\frac{1}{2}$ ✓ $2x=\pm 120^0+k.360^0$ ✓ $x=\pm 60^0+k.180^0$ ✓ $k\in Z$ ✓ -120^0 ✓ -60^0 ✓ $\cos 2x=-\frac{1}{2}$ ✓ Acute angle = 60^0= *skerphoek* ✓ both equations in $2x$ *Beide vgl in 2x* ✓ both gen solutions with $k\in Z$ *beide alg opl met* $k\in Z$ ✓ -120^0 ✓ -60^0 (6)

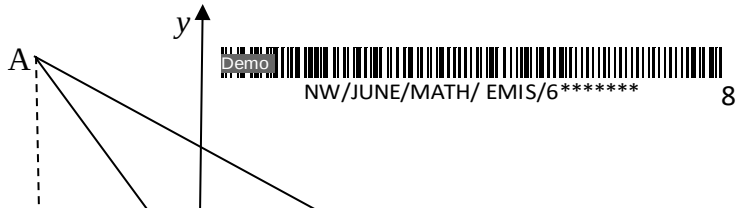
[19]

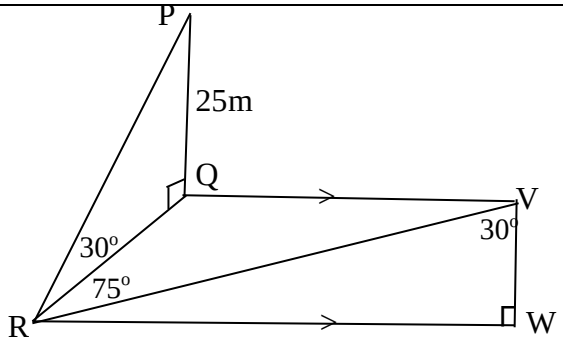
QUESTION/VRAAG 5

5.1	$a = 2$	✓✓answer *antw* (2)
5.2		✓*y*-int ✓*x*-int ✓Turning points *draaipunte* ✓Shape *vorm* (4)
5.3	2 solutions./*oplossings*	✓2 solutions *Oplossings* (1)

[7]

QUESTION /VRAAG 6

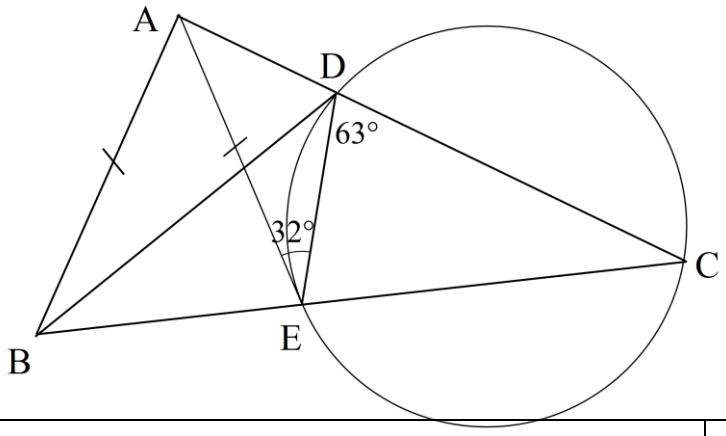
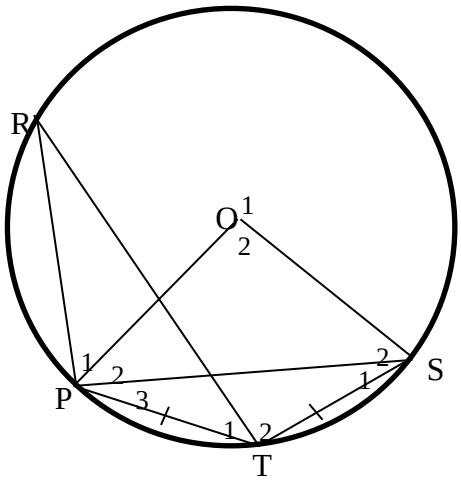


6.1	In ΔATC: $\sin C = \frac{h}{b}$ $h = b\sin C$ In ΔATB: $\sin(180^0 - B) = \frac{h}{c}$ $\sin B = \frac{h}{c}$ $h = c\sin B$ $\therefore c\sin B = b\sin C$ $\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$	✓ construction *konstruksie* ✓ $\sin C = \frac{h}{b}$ ✓ $h = b\sin C$ ✓ $\sin(180^0 - B) = \frac{h}{c}$ ✓ $h = c\sin B$ ✓ $c\sin B = b\sin C$ (6)
6.2		
6.2.1	$V\hat{R}W = 60^0$ [sum of angles of Δ/*binnehoeke van* Δ] $Q\hat{V}R = 60^0$ [Alt angles/*verwiss hoeke*; QV // RW] $R\hat{Q}V = 180^0 - 75^0 - 60^0$ [angles of ΔRQV*binnehoeke*] $= 45^0$	✓ $V\hat{R}W = 60^0$ ✓ $Q\hat{V}R = 60^0$ ✓ answer /*antw* . (3)
6.2.2	In ΔPQR: $\tan 30^0 = \frac{25}{RQ}$ $RQ = 25\sqrt{3}$ m In ΔQRV: $\frac{VR}{\sin 45^0} = \frac{25\sqrt{3}}{\sin 60^0}$ $VR = 25\sqrt{2}$ m	✓ $\tan 30^0 = \frac{25}{RQ}$ ✓ $RQ = 25\sqrt{3}$ cm ✓ $\frac{VR}{\sin 45^0}$ ✓ $\frac{25\sqrt{3}}{\sin 60^0}$ (4)

6.2.3	Area of/*van* $\Delta QRV = \frac{1}{2}\times 25\sqrt{3}\times 25\sqrt{2}\times \sin 75^0$ $= 739{,}38...$ $= 739m^2$	✓ $\frac{1}{2}\times 25\sqrt{3}\times 25\sqrt{2}\times \sin 75^0$ ✓ 739,38... ✓answer /*antw* (3)

[16]

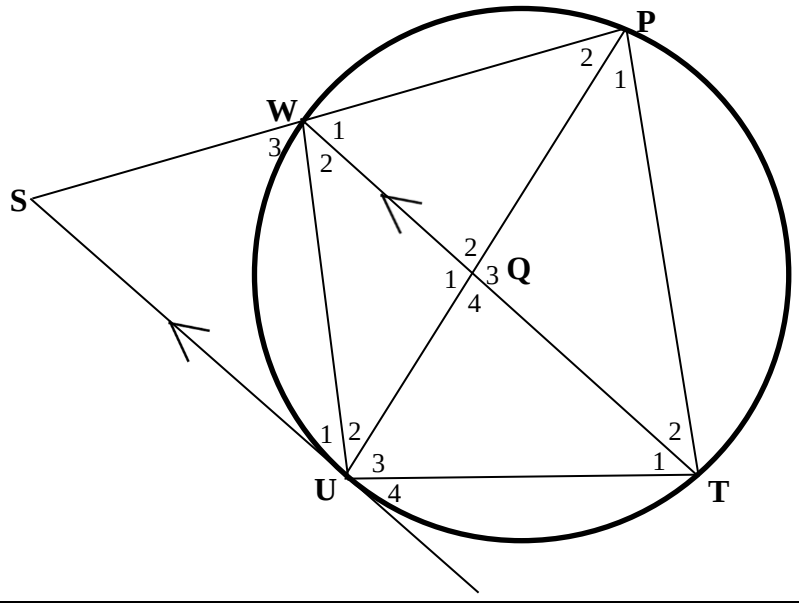
QUESTION/VRAAG 7

7.1		
7.1.1	$\hat{C} = A\hat{E}D = 32^\circ$ [tan AE and chord ED] [*raaklyn AE / koord ED*]	✓ S✓ R (2)
7.1.2	$D\hat{E}C = 85^\circ$ [sum of ∠s ΔCDE *som v binnehoeke*] $B\hat{E}A = 63^\circ$ [∠s on a str line BEC/*reguit lyn BEC*] $A\hat{B}E = A\hat{E}B = 63^\circ$[∠'s opp = sides / ∠*e teenoor = sye]*	✓ S/R ✓ S/R ✓ S/R (3)
7.2		

7.2.1	$2\hat{P}_2 + \hat{O}_2 = 180^0$ [∠s of iscos ΔPOS/ *∠e van gelykbenige ΔPOS]* $2\hat{P}_2 + 2x = 180^0$ $\hat{P}_2 = 90^0 - x$	✓ S ✓R ✓answer/*antw* (3)
7.2.2	$\hat{O}_1 = 360^0 - 2x$ [∠s around a point/ *∠e om 'n punt*] $\hat{O}_1 = 2P\hat{T}S$ [angle at centre = 2 ∠ at centre] *[midpt ∠ = 2 omtreks∠]* $2P\hat{T}S = 360^0 - 2x$ $P\hat{T}S = 180^0 - x$	✓S/R ✓S/R ✓answer /*antw* (3)
7.2.3	$2\hat{S}_1 + P\hat{T}S = 180^0$ [∠s of iscos ΔPTS/*∠e v gelykb ΔPTS*] $2\hat{S}_1 = 180^0 - 180^0 + x$ $2\hat{S}_1 = x$ $\hat{S}_1 = \frac{1}{2}x$ But $\hat{S}_1 = \hat{R}$ [subt by PT / *onderspan deur PT*] $\hat{R} = \frac{1}{2}x$	✓S/R ✓$\hat{S}_1 = \frac{1}{2}x$ ✓S/R ✓answer /*antw* (4)

[15]

QUESTION/VRAAG 8

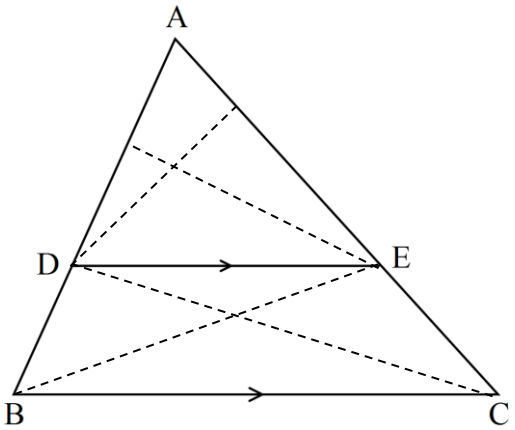


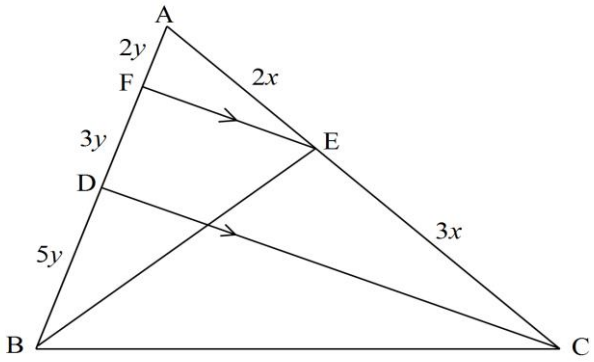
8.1	$\hat{W}_2 = \hat{U}_1 = x$ [Alt ∠s /*verwiss ∠e;* WT// US] $\hat{P}_1 = \hat{W}_2 = x$ [subt by UT/ *onderspan deur UT*] $\hat{T}_1 = \hat{W}_2 = x$ [WU = TU] $\hat{P}_2 = \hat{T}_1 = x$ [subt by WU/ *onderspan deur WU*]	✓S ✓R ✓S ✓R ✓S ✓R ✓S/R (7)
8.2	$\hat{U}_1 = \hat{T}_1$ *or* $\hat{P}_2 = x$ [proven / *bewys]* ∴ US tangent / *raaklyn* [Converse tan/chord] *[raaklyn/koord omgekeerd]*	✓S ✓R (2)
8.3	In ΔUWS and /*en* ΔPTU $\hat{U}_1 = \hat{P}_1 = x$ [Proved in / *Bewys in* 5.2.1] $\hat{W}_3 = U\hat{T}P$ [Ext ∠ of cyclic quad WUTP] *[buite ∠ van kvh WUTP]* $\hat{S} = \hat{U}_3$ [sum of ∠s of a triangle / *binne∠e v* Δ] ∴ ΔUWS /// ΔPTU [∠∠∠]	✓S/R ✓S ✓R ✓3rd ∠ / *3e ∠* or ∠∠∠ (4)

8.4	SW = 10 – 2 = 8cm
	$\frac{PQ}{5} = \frac{2}{8}$ [prop theorem / *eweredigheid* WQ // SU]
	PQ = 1,25 cm
		✓SW = 8cm
		✓S ✓R
		✓answer / *antw*
		(4)

[17]

QUESTION/VRAAG 9

9.1		✓ Construction/*konstruksie* ✓ S/R

	$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DBE} = \frac{\frac{1}{2}AD \times k}{\frac{1}{2}BD \times k}$ [same h/ *dieselfde h]* $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DBE} = \frac{AD}{BD}$ $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2}AE \times h}{\frac{1}{2}EC \times h}$ [same h/*dieselfde h*] $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DEC} = \frac{AE}{EC}$ But Area of ΔDBE = Area of ΔDEC [Δs between same // and same base / *dies basis, Δe tussen dies ‖ lyne]* $\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DEC} = \frac{AD}{BD}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓ S ✓ S/R ✓ S/R ✓ S (6)
9.2		

9.2.1	$\frac{AF}{FD} = \frac{AE}{EC} = \frac{2}{3}$ [prop theorem/*eweredigheidst* DC // FE]
	AD = DB = 5*y* [D = midpoint AB/*D = midpt AB*]
	$\frac{AF}{FD} = \frac{2y}{8y} = \frac{1}{4}$
		✓ S ✓ R
		✓ S/R
		✓ answer /*antw* (3)

9.2.2	$\frac{\text{Area of } \Delta BCE}{\text{Area of } \Delta BAE} = \frac{3}{2}$ [same height / *dies hoogte*] $\frac{\text{Area of } \Delta BAE}{\text{Area of } \Delta FAE} = \frac{10}{2}$ [same height/*dies hoogte*] $\frac{\text{Area of } \Delta BCE}{\text{Area of } \Delta BAE} \times \frac{\text{Area of } \Delta BAE}{\text{Area of } \Delta FAE} = \frac{3}{2} \times \frac{10}{2}$ $\frac{\text{Area of } \Delta BCE}{\text{Area of } \Delta FAE} = \frac{15}{2}$	✓ Area of ΔBCE = 3 ✓ Area of ΔBAE = 2 ✓ S/R ✓ Method/*metode* ✓ Answer/ *antw* (5)
		[14] TOT: 150

COGNITIVE LEVELS GRID

	CONTENT COVERED	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1.1.1			4		
1.1.2		2			
1.1.3					4
1.1.4				3	
1.1.5		1			
1.2.1		2			
1.2.2		2			
1.2.3		2			
TOTAL	**20**	**9**	**4**	**3**	**4**
2.1		4			
2.2		2			
2.3		2			
2.4					3
2.5					5
2.6			3		
2.7				3	
TOTAL	**22**	**4**	**3**	**3**	**8**
3.1			5		
3.2				6	
3.3.1		3			
3.3.2			3		
3.3.3			3		
TOTAL	**20**	**3**	**11**	**6**	**0**
4.1.1			4		
4.1.2				4	
4.2				5	
4.3			6		
TOTAL	**19**	**0**	**10**	**9**	**0**
5.1		2			
5.2			4		
5.3		1			
TOTAL	**7**	**3**	**4**	**0**	**0**
6.1			6		
6.2.1		3			
6.2.2				4	
6.2.3		3			
TOTAL	**16**	**6**	**6**	**4**	**0**
7.1.1		2			
7.1.2				3	
7.2.1			3		
7.2.2			3		
7.2.3					4
TOTAL	**15**	**2**	**6**	**3**	**4**
8.1				7	
8.2		2			
8.3			4		
8.4				4	
TOTAL	**17**	**2**	**4**	**11**	**0**
9.1			6		
9.2.1				3	
9.2.2					5
TOTAL	**14**	**0**	**6**	**3**	**5**
	GRAND TOTALS	**33**	**54**	**42**	**21**
	ACTUAL %	**22**	**36**	**28**	**14**

DISTRIBUTION OF CONTENT PAPER 2

	CONTENT	Actual Marks	Actual %
1	Statistics	20	13
2	Analytical geometry	22	15
3	Trigonometry	62	41
4	Euclidean geometry	46	31
	TOTALS	**150**	**100**