Education and Sport Development

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GRADE 12

TECHNICAL MATHEMATICS P1

MID YEAR EXAMINATION 2018

MARKS: 150
TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions, answer all questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you used to determine the answers.

3. Answer only will NOT necessarily be awarded full marks.

4. If necessary, round off answers to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. Write neatly and legibly.

QUESTION 1

1.1 Solve the following equations

1.1.1 $x(x + 3) = 4$ (3)

1.1.2 $x^2 - 6x - 2 = 0$ (Leave your answer in simplified surd form) (3)

1.2 Solve the following simultaneous equations

$2y + x = 3$ and $(x + y)(2x - y) = 0$ (6)

[12]

QUESTION 2

2.1 Solve the following inequality

$x^2 + 4x > 0$ (3)

2.2 Solve for x and y in the following equations

2.2.1 $(x + yi)(2 - i) = 8 + i$ (7)

2.2.2 $x + yi = \frac{7+i}{2-i}$ (5)

2.3 Express the complex number $z = -1 + i$ in polar form (4)

[19]

QUESTION 3

3. The roots of quadratic equation are given as $x = \frac{5 \pm \sqrt{49 - 8k}}{2}$

3.1. If $k = 5$, determine the nature of roots. (3)

3.2 Determine the value(s) of k for which roots are real. (3)

[6]

QUESTION 4

4.1 Simplify the following without the use of a calculator:

4.1.1 $\dfrac{3^{n+4} - 6.3^{n+2}}{3^{n+2}.2}$ (3)

4.1.2 $\log 15 + \log 6 - 2\log 3$ (3)

4.1.3 $\dfrac{\log 36 - \log 25}{\log 6 - \log 5}$ (3)

4.2 Solve the following equations:

4.2.1 $4^{x-1} = \sqrt{32}$ (3)

4.2.2 $2.5^{x} = 6$ (3)

4.2.3 $\log(x - 1) + \log(x + 2) - 1 = 0$ (6)

4.3 If $\log 3 = m$, determine the value of $\log\dfrac{\sqrt{3}}{3}$ in terms of m. (3)

[24]

QUESTION 5

Consider the function $g(x) = -\dfrac{4}{x} + 2$

5.1 Write down the equation of horizontal asymptote of g (1)

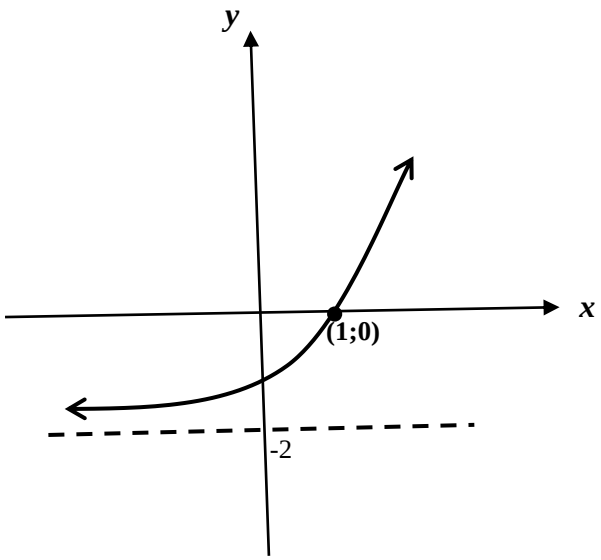
5.2 Determine the x –intercept of g (3)

5.3 Hence sketch the graph of g, showing intercept with x-axis and asymptote. (3)

[7]

QUESTION 6

6.1 The diagram below shows the graph of $f(x) = a^x + q$



6.1.1 Determine the values of a and q. (3)

6.1.2 Write down the range of f. (1)

6.1.3 Write down the equation of the asymptote of the graph of $h(x) = f(x) + 2$. (2)

6.2 Given the equation of a circle $x^2 + y^2 = 49$

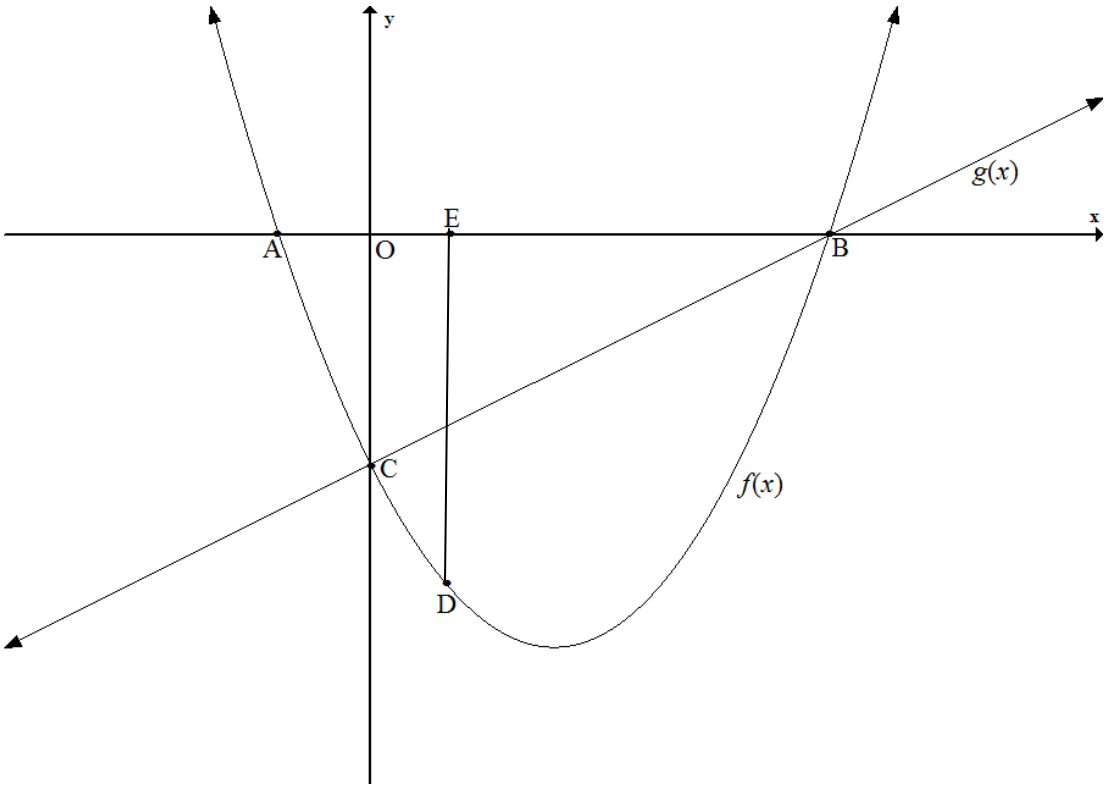
6.2.1 Write down the radius of the circle (1)

6.2.2 Write down the range of the graph $x^2 + y^2 = 49$ (2)

[9]

QUESTION 7

The figure below shows the graphs of $f(x) = 2(x - 5)(x + 1)$ and $g(x) = mx + c$.



Determine:

7.1 lengths of OA and OB (2)

7.2 the coordinates of C, the *y*-intercept of the parabola and the line (2)

7.3 the maximum length of DE where D is a variable point on the parabola between A and B (DE is perpendicular to the *x*-axis) (4)

7.4 the range of f (1)

7.5 the value(s) of x if $f(x) < g(x)$ (2)

7.6 the values of m and c (2)

7.7 the coordinates of the turning point of h, where $h(x) = -f(x)$ (2)

[15]

QUESTION 8

8.1 Find the derivative of the function $f(x) = 3x + 6$ by using the **First Principles**. (5)

8.2 Find the derivative of the following:

8.2.1 $g(x) = 7 - 5x + x^2 - 2x^3$ (3)

8.2.2 $y = \frac{7}{\sqrt{x}} - \frac{x^2}{4}$ (3)

8.3 Determine:

$Dx\left[\frac{4x+5}{2x^2}\right]$ (4)

8.4 Evaluate the following:

8.4.1 $\int (2x^2 - 1)dx$ (3)

8.4.2 $\int_{-1}^{2} (x^3 - 4x^2 + 3x)\,dx$ (4)

[22]

QUESTION 9

Given: $g(x) = x^3 + 2x^2 - x - 2$

9.1 Prove that $x - 1$ is a factor of $g(x)$. (2)

9.2 Hence, determine the x-intercepts of g. (4)

9.3 Determine the coordinates of the turning points of g. (5)

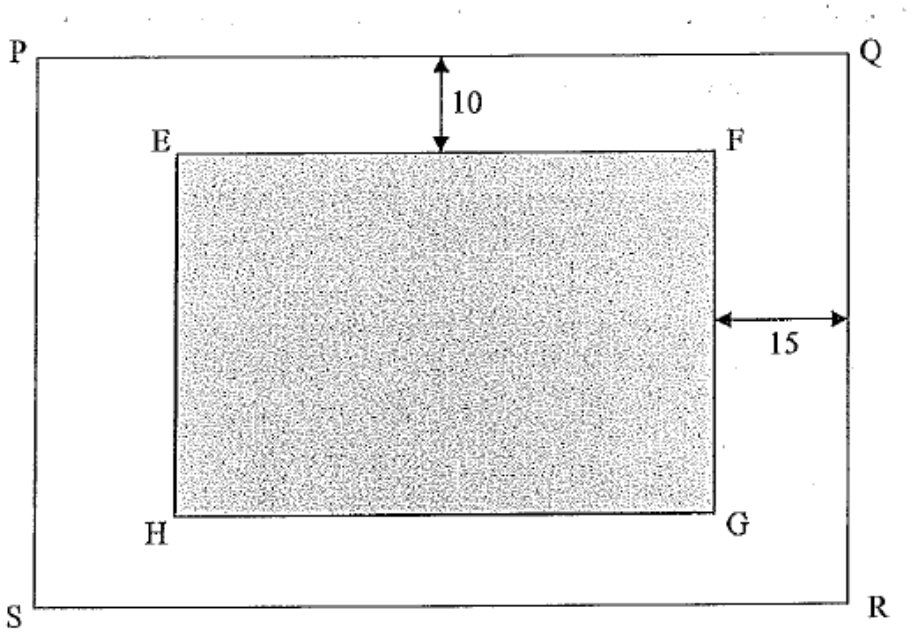
9.4 Hence, sketch the graph of g. (4)

9.5 Find the values of k for which $g(x) = k$, has only ONE real root. (2)

[17]

QUESTION 10

A page PQRS has to contain printed matter on EFGH. Open spaces of 10 mm should be left at the top and bottom of the page and 15 mm spaces on each side. The printed part must cover 5 400 mm^2.



10.1 If HG = x mm, write down the lengths of SR, EH and PS in terms of x. (3)

10.2 Hence, show that the area of PQRS is given by $6\,000 + 20x + 162\,000x^{-1}$. (2)

10.3 Determine the length of SR such that the area of the page PQRS will be a maximum. (5)

[10]

QUESTION 11

11.1 A machine has a purchase price of R10 000 and a book value of R2 000 after 5 years. Use the reducing balance method to calculate the depreciation rate. (4)

11.2 Thabo invests R120 000 with a bank for six years. The interest rate for the first two years is 8,75% p.a. compounded monthly.

It is then changed to 10,25% p.a. compounded quarterly.

What is the value of Thabo's investment at the end of six years? (5)

[9]

TOTAL:150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \ , \qquad a > 0 \ , \ a \neq 1 \ \text{and} \ b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area of } \Delta \mathrm{ABC} = \tfrac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \ ; \qquad 1 + \tan^2\theta = \sec^2\theta \ ; \qquad \cot^2\theta + 1 = \mathrm{cosec}^2\theta$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln(x) + C, \qquad x > 0 \qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$