



Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sport Ontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko

NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2 /V2 TEGNIESE WISKUNDE

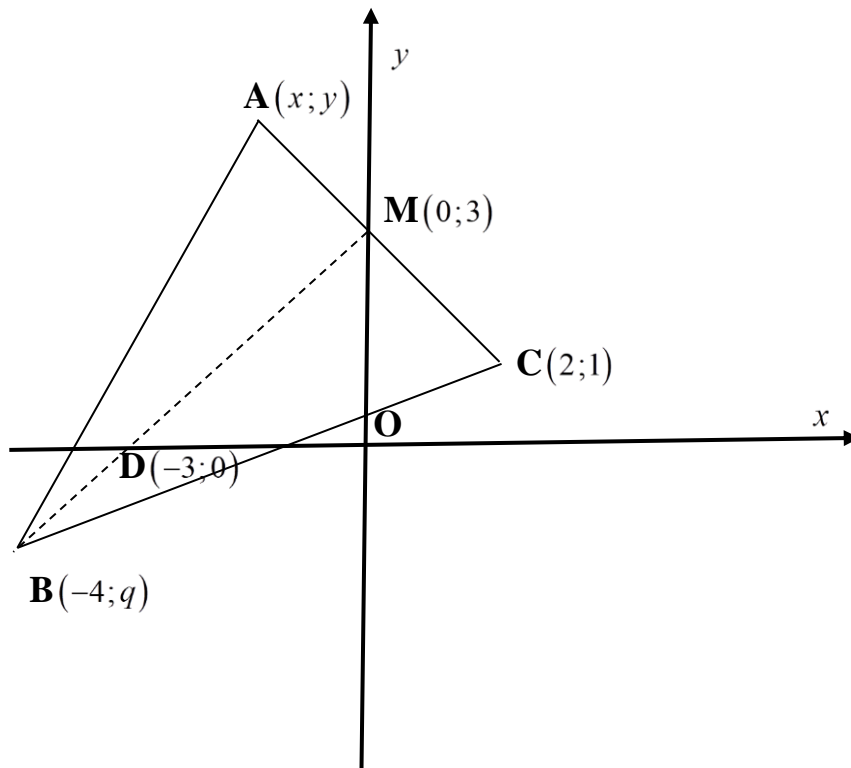
MEMORANDUM

MID-YEAR EXAMINATION 2018 HALFJAAREKSAMEN

MARKS/PUNTE: 150

This marking guidelines consists of 23 pages. / Hierdie memorandum bestaan uit 23 bladsye.

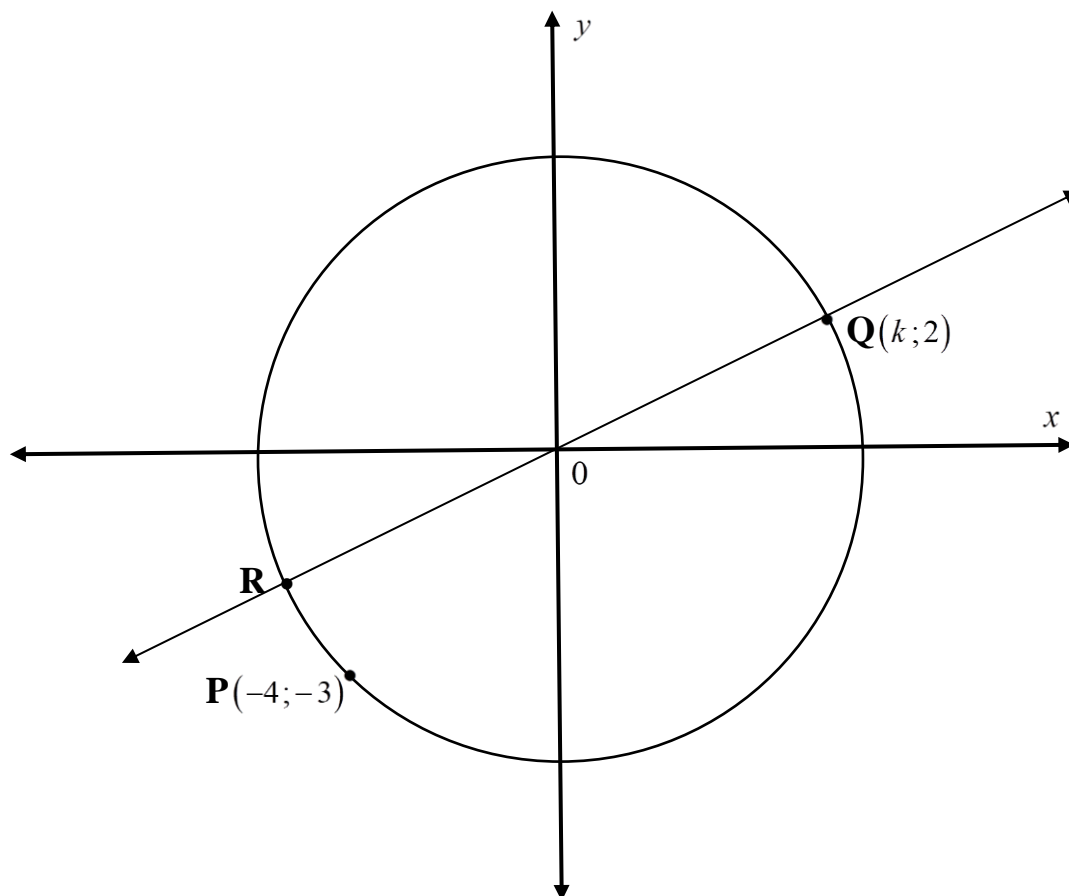
VRAAG 1



1.1	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1 - 3}{2 - 0}$ $= -1$	(2)	✓ substitution/ <i>substitusie</i> ✓ answer/ <i>antw</i>
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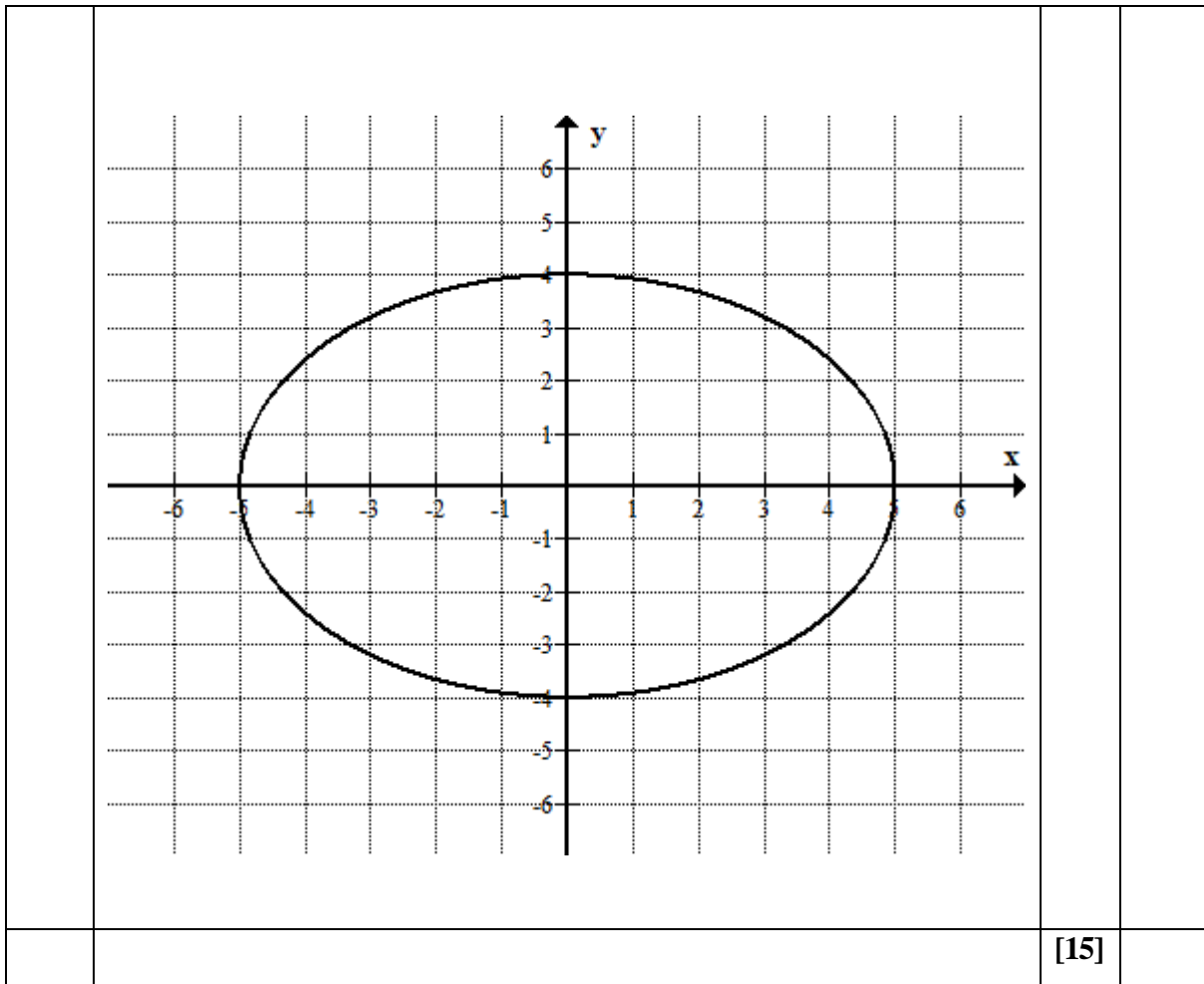
1.2	$M\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$ $= M\left(\frac{x+2}{2}; \frac{1+y}{2}\right)$ $\frac{x+2}{2} = 0 \quad \frac{1+y}{2} = 3$ $x = -2 \quad y = 5$ $A(-2;5)$	(3)	✓ setting equations / <i>vergelykings opstel</i> ✓ $x = -2$ ✓ $y = 5$
1.3	$m = \frac{0-3}{-3-0} = 1$ $y = mx + c \quad (0;3)$ $y = x + 3$	(2)	✓ $m = 1$ ✓ answer/ <i>antw</i>
1.4	$m_{AC} \times m_{MB} = -1 \times 1$ $= -1$ $\therefore AC \perp MB$	(1)	✓ $-1 \times 1 = -1$
1.5	$\tan \theta = m$ $\tan \hat{MDO} = 1$ $\hat{MDO} = \tan^{-1}(1)$ $= 45^\circ$	(2)	✓ $\tan \hat{MDO} = 1$ ✓ answer/ <i>antw</i>
1.6	$y = x + 3 \quad B(-4; q)$ $q = -4 + 3$ $= -1$ $B(-4; -1)$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $BC = \sqrt{(-4 - 2)^2 + (-1 - 1)^2}$ $= \sqrt{40}$ $= 2\sqrt{10}$	(4)	✓ $q = -4 + 3$ ✓ $q = -1$ ✓ substitution/ <i>substitusie</i> ✓ answer/ <i>antw</i>
		[14]	

VRAAG 2

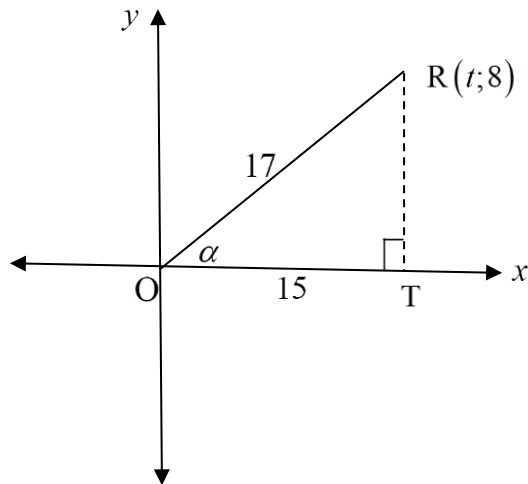


<p>2.1.1</p>	$r^2 = x^2 + y^2 \quad P(-4; -3)$ $= (-4)^2 + (-3)^2$ $= 25$ $\therefore x^2 + y^2 = 25$	<p>(2)</p>	<p>✓ $r^2 = 25$</p> <p>✓ answer/antw</p>
<p>2.1.2</p>	$x^2 + y^2 = 25 \quad Q(k; 2)$ $k^2 + 2^2 = 25$ $k = \sqrt{25 - 4}$ $= \sqrt{21} \text{ or } 4,58$	<p>(2)</p>	<p>✓ substitution/substitusie</p> <p>✓ answer/antw</p>

<p>2.1.4</p>	$m_{rad} = \frac{-3-0}{-4-0}$ $= \frac{3}{4}$ $m_{tan} \times m_{rad} = -1$ $m_{tan} \left(\frac{3}{4} \right) = -1$ $m_{tan} = -\frac{4}{3}$ $y = mx + c$ $-3 = -\frac{4}{3}(-4) + c$ $c = -\frac{16}{3} - 3$ $= -\frac{25}{3}$ $y = -\frac{4}{3}x - \frac{25}{3}$	<p>(4)</p>	$\checkmark m_{rad} = \frac{3}{4}$ $\checkmark m_{tan} = -\frac{4}{3}$ $\checkmark \text{substitution/ substitusie}$ $\checkmark \text{answer/antw}$
<p>2.2</p>	$\frac{x^2}{25} + \frac{y^2}{16} = 1$ <p>y – intercepts $x = 0$</p> $\frac{y^2}{16} = 1$ $y^2 = 16$ $y = \pm 4$ <p>x – intercepts $y = 0$</p> $\frac{x^2}{25} = 1$ $x^2 = 25$ $x = \pm 5$	<p>(3)</p>	<p>Refer to sketch <i>Verwys na skets</i></p> $\checkmark \text{shape/ vorm}$ $\checkmark \text{x – intercepts/afsnitte}$ $\checkmark \text{y – intercepts/afsnitte}$



VRAAG 3



3.1	$x^2 + y^2 = r^2 \quad [\text{Pyth}]$ $t^2 + 8^2 = 17^2$ $t = \sqrt{17^2 - 8^2}$ $= 15$	(2)	✓ substitution/ <i>substitusie</i> ✓ answer/ <i>Antw</i>
3.2	$\cos(180^\circ + \alpha)$ $= -\cos \alpha$ $= -\frac{15}{17}$	(2)	✓ $-\cos \alpha$ ✓ answer/ <i>antw</i>
3.3	$\tan \alpha = \frac{8}{15}$ $\alpha = \tan^{-1}\left(\frac{8}{15}\right)$ $= 28,07^\circ$	(2)	✓ $\frac{8}{15}$ ✓ answer/ <i>antw</i>
		[6]	

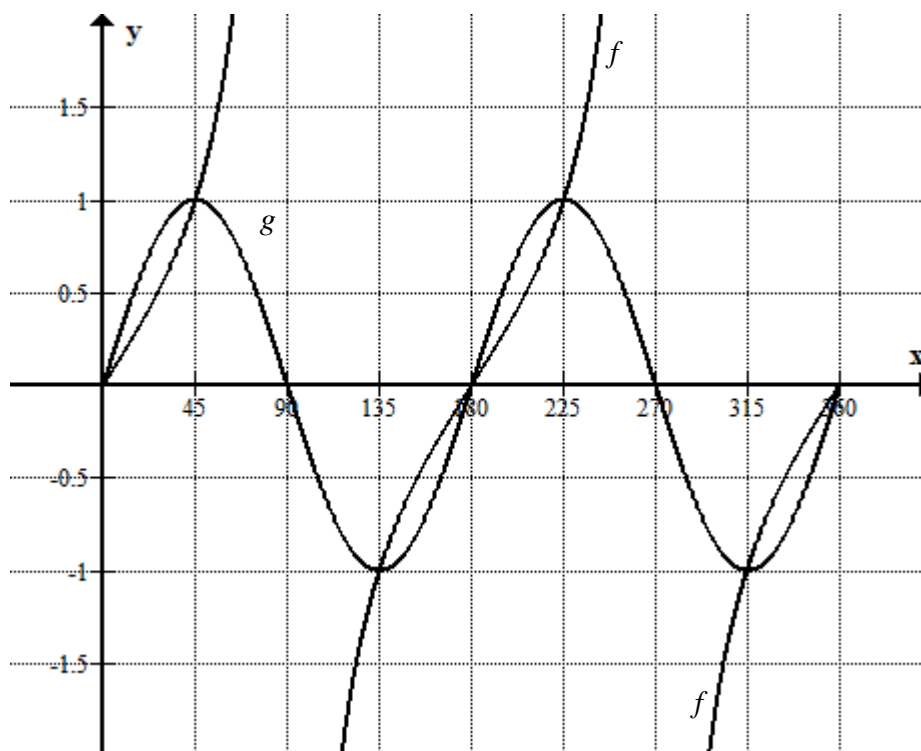
VRAAG 4

<p>4.1</p>	$\frac{\sin(-\theta)\cos(\theta+180^\circ) - \cos(90^\circ + \theta)}{\sin(540^\circ - \theta)}$ $= \frac{(-\sin\theta)(-\cos\theta) - (-\sin\theta)}{\sin\theta}$ $= \frac{\sin\theta\cos\theta + \sin\theta}{\sin\theta}$ $= \frac{\sin\theta(\cos\theta + 1)}{\sin\theta}$ $= \cos\theta + 1$	<p>(6)</p>	<p>✓ $-\sin\theta$</p> <p>✓ $-\cos\theta$</p> <p>✓ $-\sin\theta$</p> <p>✓ $\sin\theta$</p> <p>✓ common factor/<i>gemene fakt</i></p> <p>✓ answer / <i>antw</i></p>
<p>4.2</p>	$\frac{\sin(-\theta)\cos(\theta+180^\circ) - \cos(90^\circ + \theta)}{\sin(540^\circ - \theta)}$ $= \cos\theta + 1 \quad \theta = 60^\circ$ $= \cos 60^\circ + 1$ $= \frac{1}{2} + 1$ $= \frac{3}{2} \text{ or } 1,5$	<p>(2)</p>	<p>✓ $\cos 60^\circ = \frac{1}{2}$</p> <p>✓ answer / <i>antw</i></p>
<p>4.3</p>	$\text{LHS} = \frac{1}{\cos^2\theta} - \tan^2\theta$ $= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$ $= \frac{1 - \sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ $= 1$ <p>∴ LHS = RHS</p>	<p>(3)</p>	<p>✓ $\frac{\sin^2\theta}{\cos^2\theta}$</p> <p>✓ $\frac{1 - \sin^2\theta}{\cos^2\theta}$</p> <p>✓ $\cos^2\theta$</p>

<p>4.4</p>	$\frac{2 \cos 20^\circ \sin 120^\circ \sin 200^\circ}{\sin 110^\circ \sin 340^\circ}$ $= \frac{2 \cos 20^\circ \sin (180^\circ - 60^\circ) \sin (180^\circ + 20^\circ)}{\sin (90^\circ + 20^\circ) \sin (360^\circ - 20^\circ)}$ $= \frac{(2 \cos 20^\circ)(\sin 60^\circ)(-\sin 20^\circ)}{(\cos 20^\circ)(-\sin 20^\circ)}$ $= 2 \sin 60^\circ$ $= 2 \left(\frac{\sqrt{3}}{2} \right)$ $= \sqrt{3}$	<p>(5)</p>	<p>✓ $\sin 60^\circ$</p> <p>✓ $-\sin 20^\circ$</p> <p>✓ $\cos 20^\circ$</p> <p>✓ $-\sin 20^\circ$</p> <p>✓ answer/antw</p>
		<p>[16]</p>	

VRAAG 5

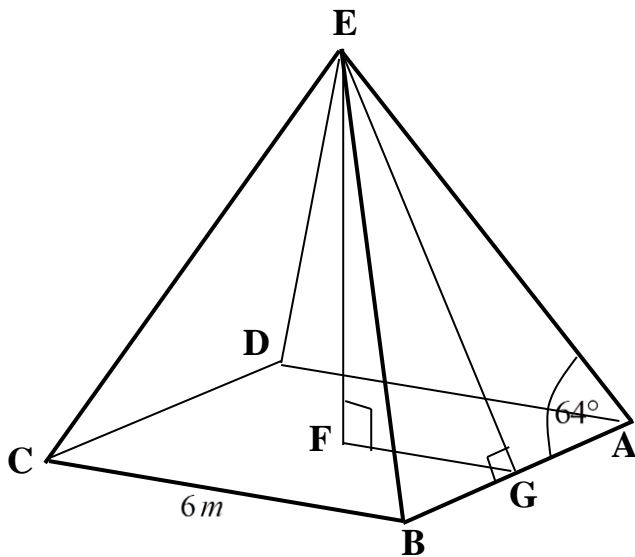
5.1



<p>5.1</p>	<p>Refer to sketch above / <i>verwys na skets hierbo</i></p>	<p>(3)</p>	<p>✓ x – intercepts / <i>afsnitte</i></p> <p>✓ min T. P / Dpt</p> <p>✓ max T. P / Dpt</p>
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5.2	$a = 1$	(1)	✓ answer/ <i>antw</i>
5.3 (a)	Period of/ <i>periode van f</i> : 180°	(1)	✓ answer/ <i>antw</i>
5.3 (b)	Period of/ <i>periode van g</i> : $\frac{360^\circ}{2} = 180^\circ$	(1)	✓ answer/ <i>antw</i>
5.4	Domain of/ <i>Definisieversameling van</i> $f : x \in [0^\circ; 360^\circ] \quad x \neq 90^\circ \quad x \neq 270^\circ$ OR/ <i>OF</i> $0^\circ \leq x < 90^\circ; \quad 90^\circ < x < 270^\circ; \quad 270^\circ < x \leq 360^\circ$	(2)	✓✓ answer / <i>antw</i> ✓✓ answer/ <i>antw</i>
5.5	$x = 45^\circ; 135^\circ$	(2)	✓ answer/ <i>antw</i> ✓ answer/ <i>antw</i>
5.6	$270^\circ < x < 315^\circ$	(4)	✓ $270^\circ < x$ ✓ $x < 315^\circ$
		[12]	

VRAAG 6

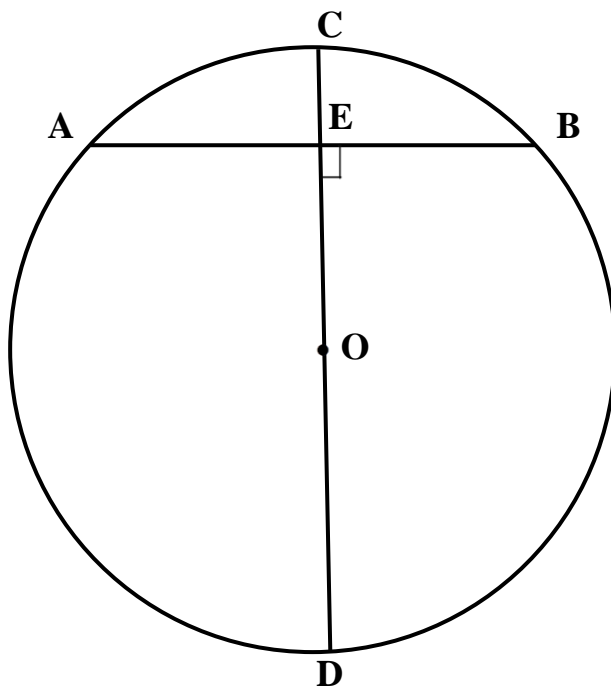


The Louvre in Paris, France

6.1	$\hat{AEG} = 26^\circ$	(1)	✓ answer/antw
6.2	$\frac{AE}{\sin 90^\circ} = \frac{AG}{\sin 26^\circ}$ $\therefore AE = \frac{3 \sin 90^\circ}{\sin 26^\circ}$ $= 6,84 \text{ m}$	(3)	✓ sine rule / <i>sin-reël</i> ✓ substitution/ <i>substitusie</i> ✓ answer/ <i>antw</i>
6.3	$\frac{EG}{\sin 64^\circ} = \frac{AG}{\sin 26^\circ}$ $\therefore EG = \frac{3 \sin 64^\circ}{\sin 26^\circ}$ $= 6,15 \text{ m}$ $EF^2 + FG^2 = EG^2 \quad [\text{Pyth}]$ $EF^2 + 3^2 = 6,15^2$ $\therefore EF = \sqrt{6,15^2 - 3^2}$ $= 5,37 \text{ m}$ <p>OR / OF</p>	(4)	✓ sine rule/ <i>sin-reël</i> ✓ 6,15 m ✓ substitution/ <i>substitusie</i> ✓ answer/ <i>answ</i>

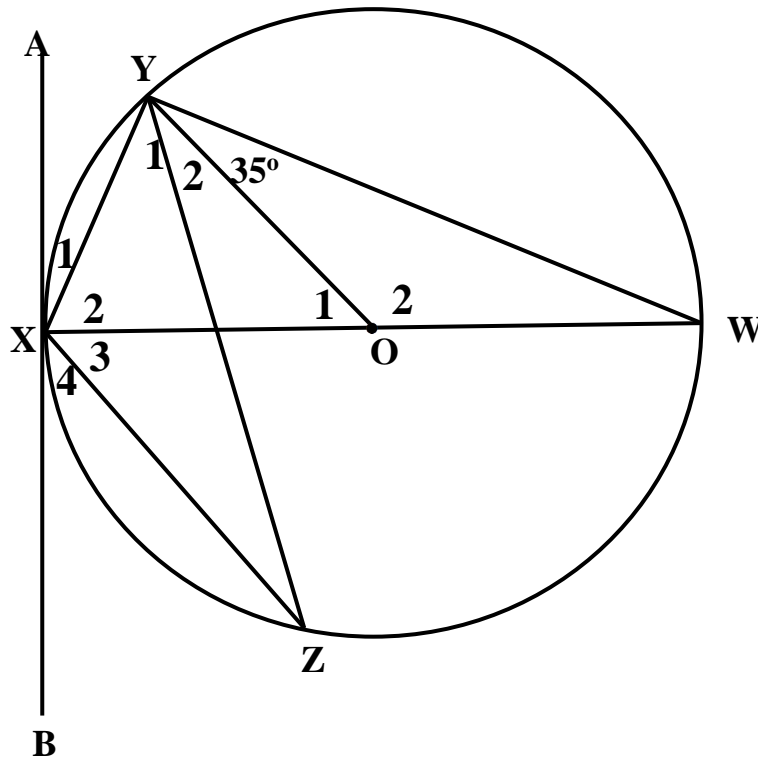
	$EG^2 = AG^2 + AE^2 - 2.AG.AE.\cos 64^\circ$ $= 3^2 + 6,84^2 - 2 \times 3 \times 6,84 \times \cos 64^\circ$ $= 37,7948.....$ $\therefore EG = 6,15 m$ <p>OR</p> $EG^2 + AG^2 = AE^2 \quad [\text{Pyth}]$ $EG^2 + 3^2 = 6,84^2$ $\therefore EG = \sqrt{6,84^2 - 3^2}$ $= 6,15 m$		<p>✓ cosine rule/<i>cos-reël</i></p> <p>✓ 6,15 m</p> <p>✓ Pythagoras</p> <p>✓ 6,15 m</p>
6.4	$V = \frac{1}{3}(\text{area van basis}) \times h$ $V = \frac{1}{3}(\text{area base}) \times h$ $= \frac{1}{3}(6 \times 6)(5,37)$ $= 64,44 m^3$	(2)	<p>✓ substitution in correct formula/ <i>substitusie in korrekte formule</i></p> <p>✓ answer/ <i>antw</i></p>
6.5	$\text{Buite - opp} = 4 \left(\frac{1}{2} \times \text{basis} \times \text{skuinshoogte} \right)$ $\text{Surface Area} = 4 \left(\frac{1}{2} \times \text{base} \times \text{slant height} \right)$ $= 4 \left(\frac{1}{2} \times 6 \times 6,15 \right)$ $= 73,80 m^2$	(3)	<p>✓ slant height / <i>skuinshoogte</i></p> <p>✓ substitution/ <i>substitusie</i></p> <p>✓ answer/ <i>antw</i></p>
		[13]	

VRAAG 7



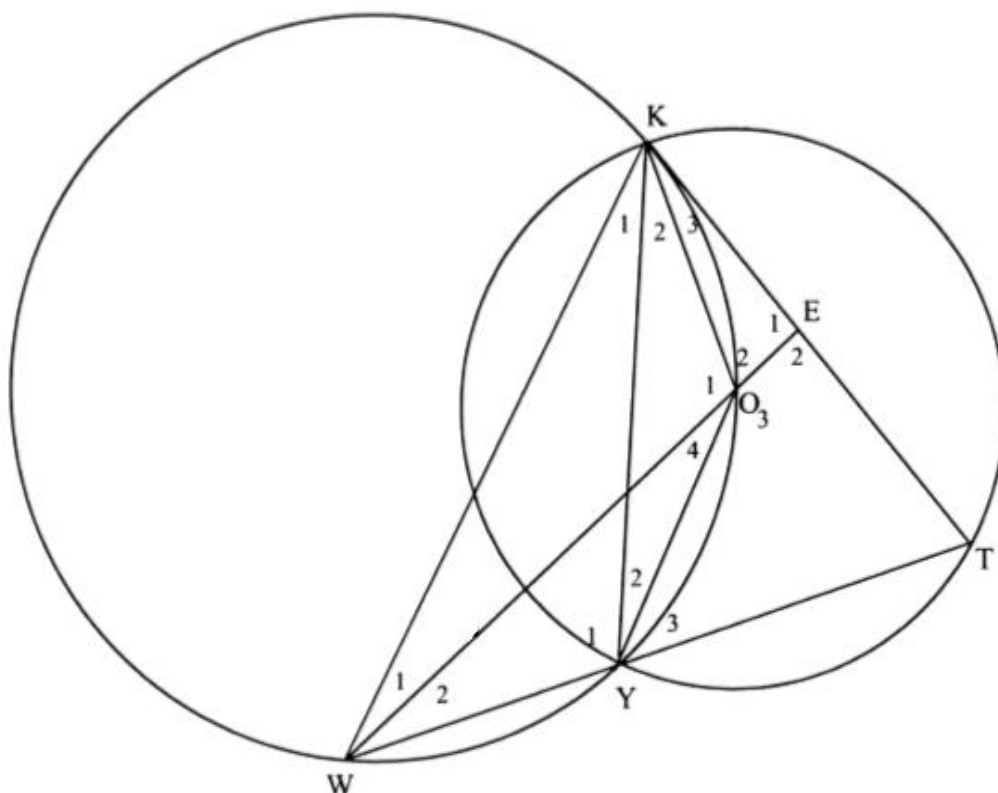
7.1	Statement (S) / <i>Bewering</i>	Reason (R) <i>Rede</i>	(3)	✓ S/ R ✓ S/R ✓ answer/ <i>antw</i>
	$EB = 3$ units / <i>eenhede</i>	Perp bisector of chord <i>radius loodreg op koord</i>		
	$OB^2 = 3^2 + 4^2$	Pythagoras		
	$OB = 5$ units			
7.2	Statement (S) / <i>Bewering</i>	Reason (R) <i>Rede</i>	(3)	✓ S/ R ✓ S/R ✓ answer/ <i>antw</i>
	$BD^2 = ED^2 + EB^2$	Pythagoras		
	$BD^2 = 9^2 + 3^2$	$ED = 4 + 5 = 9$		
	$BD = 3\sqrt{10}$ units / <i>eenhede</i>			
			[6]	

VRAAG 8



	Statement (S) <i>Bewering</i>	Reason (R) <i>Rede</i>		
8.1	$OY=OW$	both radii / <i>albei radii</i>	(3)	✓ S/ R
	$\hat{W}=O\hat{Y}W = 35^\circ$	$\angle s$ opp equal sides $\angle e$ teenoor = <i>sye</i>		✓ S/ R
	$X\hat{Z}Y=\hat{W} = 35^\circ$	$\angle s$ in same segment $\angle e$ in <i>dies segment</i>		✓ S/ R
8.2	$\hat{Y}_1=\hat{X}_4 = 30^\circ$	tan chord / <i>raaklyn/koord</i>	(3)	✓ S/ R
	$X\hat{Y}Z = 90^\circ$	\angle in semi- \odot		✓ S/ R
	$\hat{Y}_2 = 25^\circ$			✓ answer/ <i>antw</i>
8.3	$\hat{X}_4+\hat{X}_3 = 90^\circ$	tan \perp radius / <i>rklyn \perp radius</i>	(2)	✓ S/ R
	$\hat{X}_3 = 90^\circ - 30^\circ = 60^\circ$			✓ answer / <i>antw</i>
	$Z\hat{Y}W = 25^\circ + 35^\circ = 60^\circ$	Construction / <i>konstruksie</i>		
	$\hat{X}_3=Z\hat{Y}W = 60^\circ$	$\angle s$ in same segment $\angle e$ in <i>dies segment</i>		
8.4	$YW^2+XY^2=XW^2$	$X\hat{Y}W = 90^\circ$	(3)	✓ S/ R
	$YW^2 + 8^2 = (2 \times 8,5)^2$	diameter = $2 \times$ radius <i>Middellyn = 2 X radius</i>		✓ S/ R
	$YW = 15$ units / <i>eenhe</i>			✓ answer / <i>antw</i>
			[11]	

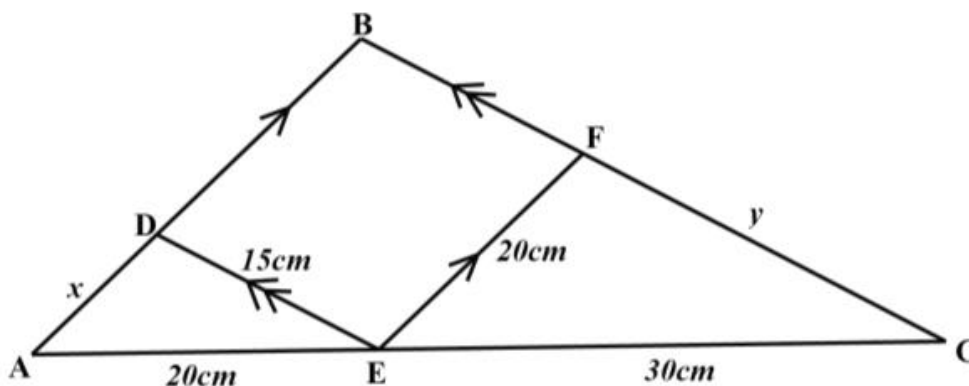
VRAAG 9



9.1	Statement (S) <i>Bewering</i>	Reason (R) <i>Rede</i>	(8)
	$\hat{W}_1 = \hat{K}_3 = 20^\circ$	tan chord theorem <i>raaklyn - koordst</i>	✓ S ✓ R
	$\hat{Y}_2 = \hat{W}_1 = 20^\circ$	$\angle s$ in the same segment $\angle e$ in dies. segment	✓ S ✓ R
	$\hat{Y}_2 = \hat{K}_3 = 20^\circ$	tan chord theorem <i>raaklyn-koordstelling</i>	✓ S ✓ R
	$\hat{K}_2 = \hat{Y}_2 = 20^\circ$	$\angle s$ opp equal sides $\angle e$ in dies segment	✓ S ✓ R
	$\hat{W}_2 = \hat{K}_2 = 20^\circ$	$\angle s$ in the same segment $\angle e$ in dies segment	✓ S ✓ R

9.2	Statement (S)/Bewering	Reason (R)/ Rede	(2)	✓ S/ R ✓ answer/antw ✓ S/ R ✓ answer/ antw
	$\hat{K}\hat{O}Y + \hat{K}_2 + \hat{Y}_2 = 180^\circ$	int \angle s Δ KOY binnehoeke		
	$\hat{K}\hat{O}Y = 180^\circ - 40^\circ = 140^\circ$			
	$\hat{K}\hat{O}Y + \hat{K}\hat{W}Y = 180^\circ$	opp \angle s of cyclic quad teenoorst \angle e kvh		
	$\hat{K}\hat{O}Y = 180^\circ - 40^\circ = 140^\circ$			
9.3	$\hat{T} = 70^\circ$	\angle at centre = $2 \times \angle$ at circumference / \angle by midpt = $2X$ by omtrek	(2)	✓ S ✓ R
9.4	$\hat{T} + \hat{W} + \hat{K} = 180^\circ$	\angle sum in Δ binnehoeke som	(5)	✓ S/ R ✓ Calculation berekening ✓ S ✓ S ✓ R
	$\hat{K} = 180^\circ - (40^\circ + 70^\circ) = 70^\circ$			
	$\hat{W}\hat{K}O = 70^\circ - 20^\circ = 50^\circ$	$\hat{K}_3 = 20^\circ$		
	$Y_3 = \hat{W}\hat{K}O = 50^\circ$	ext \angle = int opp \angle buite \angle = teenoorst binne \angle		
			[17]	

VRAAG 10



10	Statement (S)/Bewering	Reason (R)/ Rede	(6)	✓ S/ R
	BDEF is a parm	pairs of opp sides are // pare teenoorst sye is		✓ S/ R
	BF = 15 cm & BD = 20 cm	opp sides of parm = teenoorst sye v ^m =		✓ S/ R
	$\frac{x}{BD} = \frac{AE}{EC}$	prop theorem, DE // BC eweredigheidst		✓ x = 13,33 cm
	$x = \frac{20}{30} \times 20$ = 13,33 cm			✓ S/ R
	$\frac{y}{BF} = \frac{EC}{AE}$	prop theorem, AB // EF eweredigheidst		✓ y = 22,5 cm
	$y = \frac{30}{20} \times 15$ = 22,5 cm			
			[6]	

VRAAG 11



<p>11.1</p>	$n = 3700 \text{ rev/min}$ $w = ?$ $w = 2\pi n$ $= 2\pi \left(\frac{3700}{60} \right)$ $= \frac{370\pi}{3} \text{ rad.s}^{-1} \text{ or } 387,46 \text{ rad.s}^{-1}$	<p>(4)</p>	<p>✓ correct formula <i>Korrekte formule</i></p> <p>✓ substitution/ <i>substitusie</i></p> <p>✓ ÷60</p> <p>✓ answer / <i>antw</i></p>
<p>11.2</p>	$r = 0,25 \text{ m}$ $v = ?$ $v = wr$ $= 387,46 \times 0,25$ $= 96,87 \text{ m.s}^{-1}$ <p>OR / OF</p> $v = \pi Dn$ $= \pi (2 \times 0,25) \left(\frac{3700}{60} \right)$ $= \frac{185\pi}{6} \text{ m.s}^{-1} \text{ or } 96,87 \text{ m.s}^{-1}$	<p>(3)</p>	<p>✓ correct formula <i>Korrekte formule</i></p> <p>✓ substitution/ <i>substitusie</i></p> <p>✓ answer / <i>antw</i></p> <p>✓ correct formula <i>korrekte formule</i></p> <p>✓ substitution/ <i>substitusie</i></p> <p>✓ answer/ <i>antw</i></p>
		<p>[7]</p>	

VRAAG 12



12.1	$v = 6 \text{ m.s}^{-1}$	(1)	✓ answer/ <i>antw</i>
12.2	$v = wr$ $6 = w(0,36)$ $w = 16,67 \text{ rad.s}^{-1}$	(3)	✓ correct formula <i>Korrekte formule</i> ✓ substitution / <i>substitusie</i> ✓ answer / <i>antw</i>
12.3.1	$s = \frac{2\pi r}{3}$ $\theta = ?$ $s = r\theta$ $\frac{2\pi r}{3} = r\theta$ $\therefore \theta = \frac{2\pi}{3} \text{ rad}$	(4)	✓ $\frac{2\pi r}{3}$ ✓ correct formula <i>Korrekte formule</i> ✓ substitution/ <i>substitusie</i> ✓ answer / <i>antw</i>
12.3.2	Area of sector = $\frac{r^2\theta}{2}$ $\frac{(0,36)^2 \left(\frac{2\pi}{3}\right)}{2}$ $= 0,14 \text{ m}^2$	(3)	✓ correct formula <i>Korrekte formule</i> ✓ substitution / <i>substitusie</i> ✓ answer / <i>antw</i>

<p>12.4</p> $d = 72 \text{ cm}$ $x = 50 \text{ cm}$ $d = ?$ $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(72)h + (50)^2 = 0$ $4h^2 - 288h + 2500 = 0$ $h^2 - 72h + 625 = 0$ $h = \frac{-(-72) \pm \sqrt{(-72)^2 - 4(1)(625)}}{2(1)}$ $h = 36 - \sqrt{671} = 10,10 \text{ cm}$ $H = 36 + \sqrt{671} = 61,90 \text{ cm}$		<p>(5)</p>	<p>✓ substitution <i>substitusie</i></p> <p>✓ std form/ <i>std-vorm</i></p> <p>✓ quad formula <i>Kwadratiese formule</i></p> <p>✓ 10,10 cm</p> <p>✓ 61,90 cm</p>
		<p>[16]</p>	

VRAAG 13

Gautrain



<p>13.1</p>	$t = 35 \text{ min}$ $v = 140 \text{ km / h}$ $s = ?$ $v = \frac{s}{t}$ $\frac{140}{60} = \frac{s}{35}$ $\therefore s = \frac{140}{60} \times 35$ $= 81,67 \text{ km}$	<p>(3)</p>	<p>✓ formula/ <i>formule</i></p> <p>✓ substitution/ <i>substitusie</i></p> <p>✓ answer/ <i>antw</i></p>
<p>13.2.1</p>	$s = 8 \text{ km}$ $r = 9 \text{ km}$ $t = 5 \text{ min}$ $\theta = ?$ $s = r\theta$ $8 = (9)\theta$ $\therefore \theta = 0,89 \text{ rad}$	<p>(2)</p>	<p>✓ substitution/ <i>substitusie</i></p> <p>✓ answer/ <i>antw</i></p>

13.2.2	$w = \frac{\theta}{t}$ $= 0,89 \div \frac{5}{60}$ $= 10,68 \text{ rad} \cdot \text{s}^{-1}$ $v = rw$ $= 9 \times 10,68$ $= 96,12 \text{ km} / \text{h}$	(4)	✓ substitution / <i>substitusie</i> ✓ 10,68 ✓ substitution / <i>substitusie</i> ✓ answer / <i>antw</i>
13.3	Total Costs = $41 \times 2 \times 20$ = R1640	(2)	✓ method / <i>metode</i> ✓ answer / <i>antw</i>
		[11]	
	TOTAL/ TOTAAL	[150]	